TRİGONOMETRİK DENKLEMLER

1) $\begin{matrix} sinx = sin\alpha \\ sinx = -sin\alpha \end{matrix}$ denklemlerinin çözümü

$sinx = sin\alpha \Leftrightarrow x = \alpha + k.2\pi$ veya $x = (\pi - \alpha) + k.2\pi, k\epsilon\mathbb{Z}$

$sinx = -sin\alpha \Leftrightarrow x = (\pi + \alpha) + k.2\pi$ veya $x = (-\alpha) + k.2\pi = (2\pi - \alpha) + k.2\pi$

ÖRNEK: $sinx = \frac{1}{2}$ denkleminin çözüm kümesini bulunuz.

$[0, \frac{\pi}{2})$ aralığında sinüsü $\frac{1}{2}$ olan açı 30^o dir.

$sinx = sin\frac{\pi}{6} \Leftrightarrow x = \frac{\pi}{6} + k.2\pi$ veya $x = \left(\pi - \frac{\pi}{6}\right) + k.2\pi$

$ÇK = \{ x \,|x = \frac{\pi}{6} + k.2\pi \vee x = \frac{5\pi}{6} + k.2\pi, k \in \mathbb{Z} \}$

ÖRNEK: $sinx = -\frac{\sqrt{3}}{2}$ denkleminin çözüm kümesini bulunuz.

$[0, \frac{\pi}{2})$ aralığında sinüsü $\frac{\sqrt{3}}{2}$ olan açı 60^o dir yani $\frac{\pi}{3}$.

$$sinx = -\frac{\sqrt{3}}{2} \Leftrightarrow sinx = -\sin\frac{\pi}{3} \Leftrightarrow x = \left(\pi + \frac{\pi}{3}\right) + k.2\pi = \frac{4\pi}{3} + k.2\pi \text{ veya}$$

$$\Leftrightarrow x = -\frac{\pi}{3} + k.2\pi = \left(2\pi - \frac{\pi}{3}\right) + k.2\pi = \frac{5\pi}{3} + k.2\pi, k \in \mathbb{Z}$$

$ÇK = \{ x \,|x = \frac{4\pi}{3} + k.2\pi \vee x = \frac{5\pi}{3} + k.2\pi, k \in \mathbb{Z} \}$

ÖRNEK: $sin3x = \sin\left(2x + \frac{\pi}{3}\right)$ denkleminin çözüm kümesini bulunuz.

$2x + \frac{\pi}{3} = \alpha$ olsun.

$sin3x = sin\alpha \Leftrightarrow 3x = \alpha + k.2\pi \;\vee\; 3x = (\pi - \alpha) + k.2\pi, k \in \mathbb{Z}$

$$3x = 2x + \frac{\pi}{3} + k.2\pi \;\vee\; 3x = \pi - 2x - \frac{\pi}{3} + k.2\pi, k \in \mathbb{Z}$$

$$x = \frac{\pi}{3} + k.2\pi \text{ veya } 5x = \frac{2\pi}{3} + k.2\pi$$

$$x = \frac{\pi}{3} + k.2\pi \text{ veya } x = \frac{2\pi}{15} + k.\frac{2\pi}{5}, \; k \in \mathbb{Z}$$

$ÇK = \{ x \,|x = \frac{\pi}{3} + k.2\pi \vee x = \frac{2\pi}{15} + k.\frac{2\pi}{5}, k \in \mathbb{Z} \}$

ÖRNEK: $2sin^2x - 5sinx + 2 = 0$ denkleminin çözüm kümesini bulunuz.

$sinx = t$ olsun. $2t^2 - 5t + 2 = 0 \Leftrightarrow (2t-1)(t-2) = 0 \Leftrightarrow t = \frac{1}{2}$ veya $t = 2$

$sinx = 2 \Longrightarrow ÇK = \emptyset$ (Çünkü $-1 \leq sinx \leq 1, \forall x \in \mathbb{R}$)

$sinx = \frac{1}{2} \Longrightarrow$ 1. Örnekten, $ÇK = \{ x \mid x = \frac{\pi}{6} + k.2\pi \vee x = \frac{5\pi}{6} + k.2\pi, k \in \mathbb{Z} \}$

2) $cosx = cos\alpha$, $cosx = -cos\alpha$ denklemlerinin çözümü

$cosx = cos\alpha \Leftrightarrow x = \alpha + k.2\pi$ veya $x = (-\alpha) + k.2\pi, k\epsilon\mathbb{Z}$

$\Leftrightarrow x = \alpha + k.2\pi$ veya $x = (2\pi - \alpha) + k.2\pi$

$cosx = -cos\alpha \Leftrightarrow x = (\pi - \alpha) + k.2\pi$ veya $x = (\pi + \alpha) + k.2\pi, k\epsilon\mathbb{Z}$

ÖRNEK: $cosx = -1$ ise $ÇK =$?

π nin tek katlarında cosinüs -1 olduğundan $x = (2k+1)\pi, k \in \mathbb{Z}$

$ÇK = \{ x \mid x = (2k+1)\pi, k \in \mathbb{Z} \}$

ÖRNEK: $2cos^2x - 5cosx + 2 = 0$ ise $ÇK =$?

$(2cosx - 1)(cosx - 2) = 0 \Leftrightarrow cosx = \frac{1}{2}$ veya $cosx = 2$

$\forall x \in \mathbb{R}\ -1 \leq cosx \leq 1$ olduğundan cosx=2 denklemi için $ÇK = \emptyset$

$cosx = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{3} + k.2\pi \vee x = \left(2\pi - \frac{\pi}{3}\right) + k.2\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{3} + k.2\pi \vee x = \frac{5\pi}{3} + k.2\pi, k \in \mathbb{Z}$

$ÇK = \{ x \mid x = \frac{\pi}{3} + k.2\pi \vee x = \frac{5\pi}{3} + k.2\pi, k \in \mathbb{Z} \}$

ÖRNEK: $cos2x = sin\frac{3\pi}{8}$ ise $ÇK =$?

$sin\alpha = cos\left(\frac{\pi}{2} - \alpha\right)$ olduğundan $\sin\frac{3\pi}{8} = cos\left(\frac{\pi}{2} - \frac{3\pi}{8}\right) = cos\frac{\pi}{8}$

$cos2x = cos\frac{\pi}{8} \Leftrightarrow 2x = \frac{\pi}{8} + k.2\pi$ veya $2x = \left(2\pi - \frac{\pi}{8}\right) + k.2\pi$

$\Leftrightarrow x = \frac{\pi}{16} + k\pi$ veya $x = \frac{15}{16} + k\pi\ , k \in \mathbb{Z}$

$ÇK = \{ x \mid x = \frac{\pi}{16} + k\pi \vee x = \frac{15\pi}{16} + k\pi \ , k \in \mathbb{Z} \}$

ÖRNEK: $2cos2x + sinx + 3 = 0$ denkleminin $[0, 2\pi]$ de $ÇK$ =?

$2cos2x + sinx + 3 = 0 \Longrightarrow 2(1 - 2sin^2x) + sinx + 3 = 0, \quad sinx = t \Longrightarrow 4sin^2x - sinx - 5 = 4t^2 - t - 5 = 0 \Longrightarrow t_{1,2} = \frac{1 \pm 9}{8} \Longrightarrow t_1 = \frac{5}{4}, t_2 = -1$

$sinx = \frac{5}{4}$ için $ÇK = \emptyset$

$sinx = -1$ için $x = \frac{3\pi}{2}$, $ÇK = \left\{\frac{3\pi}{2}\right\}$

3) $\left.\begin{matrix} tanx = tan\alpha \\ cotx = -cot\alpha \end{matrix}\right\} \Leftrightarrow x = \alpha + k\pi, k \in \mathbb{Z}$

ÖRNEK: $tanx = cotx \quad ÇK$ =?

$cotx = tan\left(\frac{\pi}{2} - x\right), \frac{\pi}{2} - x = \alpha$ denirse

$tanx = tan\alpha \Leftrightarrow x = \alpha + k\pi, k \in \mathbb{Z} \Leftrightarrow x = \left(\frac{\pi}{2} - x\right) + k\pi \Longrightarrow 2x = \frac{\pi}{2} + k\pi \Longrightarrow x = \frac{\pi}{4} + k\frac{\pi}{2}$

$ÇK = \{ x \mid x = \frac{\pi}{4} + k\frac{\pi}{2} \ , k \in \mathbb{Z} \}$

ÖRNEK: $tan2x . cot\left(3x - \frac{\pi}{3}\right) = 1 \quad ÇK$ =?

$tan\left(3x - \frac{\pi}{3}\right) cot\left(3x - \frac{\pi}{3}\right) = 1 \Longrightarrow cot\left(3x - \frac{\pi}{3}\right) = {}^{1}\!\Big/_{tan\left(3x - \frac{\pi}{3}\right)}$ olur.

$$(tan2x).\left({}^{1}\!\Big/_{tan\left(3x - \frac{\pi}{3}\right)}\right) = 1 \Longrightarrow tan\left(3x - \frac{\pi}{3}\right) = \tan 2x \Longrightarrow 3x - \frac{\pi}{3} = 2x + k\pi \Longrightarrow$$

$$x = \frac{\pi}{3} + k\pi$$

$ÇK = \{ x \mid x = \frac{\pi}{3} + k\pi \ , k \in \mathbb{Z} \}$

4) $\left.\begin{matrix} tanx = -tan\alpha \\ cotx = -cot\alpha \end{matrix}\right\} \Leftrightarrow x = (\pi - \alpha) + k\pi, k \in \mathbb{Z}$

ÖRNEK: $tan4x = -\frac{\sqrt{3}}{3}$ $ÇK = ?$

$tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$ $tan4x = -tan\frac{\pi}{6} \Longrightarrow 4x = \left(\pi - \frac{\pi}{6}\right) + k\pi = \frac{5\pi}{6} + k\pi$

$ÇK = \{ x \,|x = \frac{5\pi}{24} + \mathrm{k}\frac{\pi}{4} \, , k \in \mathbb{Z} \}$

5) *$acosx + bsinx = c$ denkleminin çözümü*

Eşitliğin her iki yanı a ile bölünerek $cosx$ in katsayısı 1 yapılır ve sinx in katsayısı olarak bulunan $\frac{b}{a}$ yerine $tan\alpha$ yazılır. Gerekli işlemler yapılarak denklem çözülür.

$$cosx + \frac{b}{a}sinx = \frac{c}{a}$$

$$cosx + tan\alpha sinx = \frac{c}{a}$$

ÖRNEK: $\sqrt{3}cosx + 3sinx = \sqrt{6}$ denkleminin $ÇK = ?$

$$\frac{\sqrt{3}}{\sqrt{3}}cosx + \frac{3}{\sqrt{3}}sinx = \frac{\sqrt{6}}{\sqrt{3}}$$

$$cosx + \sqrt{3}sinx = \sqrt{2}$$

$\sqrt{3}$ yerine $tan\frac{\pi}{3}$ yazılırsa

$$cosx + \tan\frac{\pi}{3}sinx = \sqrt{2} \Longrightarrow cosx + \frac{sin\frac{\pi}{3}}{cos\frac{\pi}{3}}sinx = \sqrt{2}$$

$$cosx\,cos\frac{\pi}{3} + sin\frac{\pi}{3}sinx = \sqrt{2}cos\frac{\pi}{3}$$

$$cos\left(x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} = cos\frac{\pi}{4}$$

$\Leftrightarrow x - \frac{\pi}{3} = \frac{\pi}{4} + k.2\pi$ veya $x - \frac{\pi}{3} = 2\pi - \frac{\pi}{4} + k.2\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{7\pi}{12} + k.2\pi$ veya $x = \frac{25}{12} + k.2\pi$

$\Leftrightarrow x = \frac{7\pi}{12} + k.2\pi$ veya $x = \frac{\pi}{12} + 2\pi + k.2\pi$

$\Leftrightarrow x = \frac{7\pi}{12} + k.2\pi$ veya $x = \frac{\pi}{12} + k^*.2\pi$

$ÇK = \{ x \mid x = \frac{7\pi}{12} + k.2\pi$ veya $x = \frac{\pi}{12} + k.2\pi \ , k \in \mathbb{Z} \}$

ÖRNEK: $tanx + cotx - \frac{4}{\sqrt{3}} = 0 \quad ÇK = ?$

$tanx + cotx - \frac{4}{\sqrt{3}} = 0 \Longrightarrow tanx + cotx = \frac{4}{\sqrt{3}} \Longrightarrow \frac{sinx}{cosx} + \frac{cosx}{sinx} = \frac{4}{\sqrt{3}} \Longrightarrow \frac{sin^2x + cos^2x}{sinxcosx} = \frac{4}{\sqrt{3}}$

$$\Longrightarrow \frac{1}{\frac{1}{2}sin2x} = \frac{4}{\sqrt{3}} \Longrightarrow 2sin2x = \sqrt{3} \Longrightarrow sin2x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$$

$$\vdots$$

$\Longrightarrow 2x = \frac{\pi}{3} + 2k\pi$ veya $2x = (\pi - \frac{\pi}{3}) + 2k\pi$