s.s. indirgeme formülleri:

$n \geq 2$ olmak üzere $\int \cos^n x\,dx$, $\int \sin^n x\,dx$, $\int \tan^n x\,dx$, $\int \sec^n x\,dx$ ve bunlar gibi bazı integraller için indirgeme formülü elde edilebilir. Bu formüller genellikle kısmi integrasyon ya da değişken dönüşümü ile bulunabilir.

Örnek: 1) $\int \cos^n x\,dx$ integrali için bir indirgeme formülü bulalım:

$I_n = \int \cos^n x\,dx = \int \cos^{n-1} x \cdot \cos x\,dx$ $\left(\begin{array}{l} u = \cos^{n-1} x \Rightarrow du = -(n-1)\cdot\cos^{n-2} x \cdot \sin x\,dx \\ dv = \cos x \Rightarrow v = \sin x \end{array} \right)$

$$I_n = \int \cos^n x\,dx = \sin x \cdot \cos^{n-1} x + (n-1)\cdot\int \cos^{n-2} x \cdot \sin^2 x\,dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1)\cdot\int \cos^{n-2} x \cdot (1-\cos^2 x)\,dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1)\cdot\int \cos^{n-2} x\,dx - (n-1)\cdot\int \cos^n(x)\,dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1)\cdot I_{n-2} - (n-1) I_n$$

$$I_n = \boxed{\frac{1}{n}\sin x \cdot \cos^{n-1} x + \frac{n-1}{n}\cdot I_{n-2} = \int \cos^n x\,dx}$$ cos için indirgeme formülü

2) $\int \tan^n x\,dx = \int \tan^{n-2} x \cdot \tan^2 x\,dx = \int \tan^{n-2} x \cdot (\sec^2 x - 1)\,dx$

$= \int \tan^{n-2} x \cdot \sec^2 x\,dx - \int \tan^{n-2} x\,dx$ $\left(\begin{array}{l} u = \tan x \\ du = \sec^2 x\,dx \end{array} \right)$

$$\boxed{\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - I_{n-2}}$$ indirgeme formülü bulunur.

$\left(\begin{array}{l} u = \sec^{n-2} x \Rightarrow du = (n-2)\cdot\sec^{n-2} x \cdot \tan x\,dx \\ dv = \sec^2 x\,dx \Rightarrow v = \tan x \end{array} \right)$

3) $\int \sec^n x\,dx = \int \sec^{n-2} x \cdot \sec^2 x\,dx$

$$= \sec^{n-2} x \cdot \tan x - (n-2)\cdot\int \sec^{n-2} x \cdot \tan^2 x\,dx$$

$$= \sec^{n-2} x \cdot \tan x - (n-2)\cdot\int \sec^{n-2} x \cdot (\sec^2 x - 1)\,dx$$

$$= \sec^{n-2} x \cdot \tan x - (n-2)\cdot\int \sec^n x\,dx + (n-2)\int \sec^{n-2} x\,dx$$

$$= \sec^{n-2} x \tan x - (n-2)\cdot\int \sec^{n-2} x\,dx$$

$$(n-1)\cdot\int \sec^n x\,dx = \sec^{n-2} x \cdot \tan x + (n-2)\cdot\int \sec^{n-2} x\,dx$$

$$\boxed{I_n = \int \sec^n x\,dx = \frac{1}{n-1}\cdot\sec^{n-2} x \cdot \tan x + \frac{n-2}{n-1}\cdot I_{n-2}}$$

4) $\int \sin^n x\,dx = -\frac{1}{n}\cdot\cos x \cdot \sin^{n-1} x + \frac{n-1}{n}\cdot\int \sin^{n-2} x\,dx$

$\int \cot^n x\,dx = -\frac{1}{n-1}\cot^{n-1} x - \int \cot^{n-2} x\,dx$

$\int \operatorname{cosec}^n x\,dx = \frac{-1}{n-1}\cdot\cot x \cdot \csc^{n-2} x + \frac{n-2}{n-1}\cdot\int \csc^{n-2} x\,dx$

indirgeme formüllerini elde ediniz.

Çözülmüş örnekler:

1) $\int \cos^3 x\,dx = ?$ $\int \cos^4 x\,dx = ?$

indirgeme formülünde sırasıyla $n=3$, $n=4$ yazılır.

$$\int \cos^3 x\,dx = \frac{1}{3}\cos^2 x.\sin x + \frac{2}{3}.\int \cos x\,dx$$
$$= \frac{1}{3}\cos^2 x.\sin x + \frac{2}{3}\sin x + c$$

$$\int \cos^4 x\,dx = \frac{1}{4}\cos^3 x.\sin x + \frac{3}{4}\int \cos^2 x\,dx$$
$$\overset{(n=2)}{=} \frac{1}{4}\cos^3 x.\sin x + \frac{3}{4}.\left[\frac{1}{2}\cos x.\sin x + \frac{1}{2}\int dx\right] + c$$
$$= \frac{1}{4}\cos^3 x.\sin x + \frac{3}{8}\cos x \sin x + \frac{3}{8}x + c$$

2) $\int \tan^5 x\,dx = ?$ $(n=5)$

$$\int \tan^5 x\,dx = \frac{\tan^4 x}{4} - \int \tan^3 x\,dx \overset{(n=3)}{=} \frac{\tan^4 x}{4} - \left[\frac{\tan^2 x}{2} - \int \tan x\,dx\right]$$
$$= \frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x - \ln|\cos x| + C$$

3) $\int \sec^6 x\,dx = \int \sec^4 x.\sec^2 x\,dx = \int (\sec^2 x)^2.\sec^2 x\,dx =$
$$= \int (1+\tan^2 x)^2.\sec^2 x\,dx \qquad u=\tan x \Rightarrow du = \sec^2 x\,dx$$
$$= \int (u^2+1)^2.du = \int (u^4+2u^2+1)du$$
$$= \frac{1}{5}\tan^5 x + \frac{2}{3}\tan^3 x + \tan x + C$$

4) $\int \tan^2 x.\sec^4 x\,dx = \int \tan^2 x.\sec^2 x.\sec^2 x\,dx = \int \tan^2 x.(\tan^2 x+1).\sec^2 x\,dx$
$$= \int u^2.(u^2+1).du = \int (u^4+u^2)\,du = \frac{u^5}{5} + \frac{u^3}{3} + 1$$
$$= \frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + 1$$

5) $\int \tan^2 x \sec x\,dx = \int (\sec^2 x - 1).\sec x\,dx = \int \sec^3 x\,dx - \int \sec x\,dx$

6) $\int \tan^3 x.\sec^3 x\,dx = \int \tan^2 x.\sec^2 x.\tan x.\sec x.dx$ $\left(\begin{array}{l} u=\sec x \\ du=\sec x \tan x\,dx \end{array}\right)$
$$= \int (\sec^2 x - 1).\sec^2 x.\sec x.\tan x\,dx$$
$$= \int (u^2-1).u^2.du = \int u^4\,du - \int u^2\,du$$
$$= \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x\,dx$$

ödev problemler: (S.1, S.2, S.3, S.4, S.5)

1) $\int \frac{3x}{4x-1}\,dx$ 2) $\int \frac{1}{x^2(1-5x)}\,dx$ 3) $\int \frac{x}{\sqrt{2-x}}$ 4) $\int \sin 3x \cdot \sin 2x\,dx$

5) $\int \sin 2x \cdot \cos 5x\,dx$ 6) $\int x^3 \cdot \ln x\,dx$ 7) $\int \frac{\ln x}{\sqrt{x}}\,dx$

8) $\int e^{-2x} \sin 3x\,dx$ 9) $\int e^{x} \cdot \cos 2x\,dx$ 10) $\int \frac{e^{4x}}{(4-3 \cdot e^{2x})^2}\,dx$ $(u=e^{2x})$

11) $\int \frac{\cos 2x}{(\sin 2x) \cdot (3-\sin 2x)}\,dx$, $u=\sin 2x$ 12) $\int \frac{1}{\sqrt{x} \cdot (9x+4)}\,dx$, $u=3\sqrt{x}$

13) $\int \frac{\cos 4x}{9+\sin^2 4x}\,dx$, $u=\sin 4x$ 14) $\int \frac{\sin^2(\ln x)}{x}\,dx$, $u=\ln x$

15) $\int \frac{\sin 3x}{(\cos 3x) \cdot (\cos 3x+1)^2}\,dx$ 16) $\int \frac{e^x}{3-4 \cdot e^{2x}}\,dx$ 17) $\int e^x \cdot \sqrt{3-4 \cdot e^{2x}}\,dx$

18) $\int (\ln x)^2\,dx$ 19) $\int x \cdot \sec^2 x\,dx$ 20) $\int e^{-3\theta} \cdot \sin 3\theta\,d\theta$

21) $\int x^n e^x\,dx = x^n e^x - n \cdot \int x^{n-1} e^x\,dx$ old. göst.

22) $\int \cos^4\left(\frac{x}{4}\right)dx$ 23) $\int \cos^{1/5} x \sin x\,dx$ 24) $\int \sin^5\theta \cdot \cos^4\theta\,d\theta$

25) $\int \sin x \cdot \cos\left(\frac{x}{2}\right)dx$ 26) $\int \sec^2(3x+1)\,dx$ 27) $\int \tan^2(1-2x) \cdot \sec(1-2x)\,dx$