Uygulama 10

1-) $\int \operatorname{cosec} x \cot x \, dx = \int \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} \, dx$

$$= \int \frac{\cos x}{\sin^2 x} \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \frac{du}{u^2}$$

$$= -\frac{1}{u} + C$$

$$= -\frac{1}{\sin x} + C$$

(2018 Bütünleme)

2-) $\int \sin 7x \cdot \sin 5x \, dx = \frac{1}{2} \int (\cos 2x - \cos 12x) \, dx$

$$= \frac{1}{2} \left(\frac{\sin 2x}{2} - \frac{\sin 12x}{12} \right) + C$$

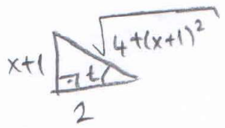
$$= \frac{1}{4} \sin 2x - \frac{1}{24} \sin 12x + C$$

3-) $\int \frac{3x+2}{x^2+2x+5}\,dx = \int \frac{3x+2}{(x+1)^2+2^2}\,dx$

$x+1 = 2\tan t$

$dx = 2\sec^2 t\,dt$

$\tan t = \frac{x+1}{2}$



$= \int \frac{3(2\tan t - 1) + 2}{4(\cancel{\tan^2 t + 1})}\, 2\cancel{\sec^2} t\,dt$

$= \frac{1}{2}\int 6\tan t - 1\,dt$

$= 3\int \tan t\,dt - \frac{1}{2}\int dt$

$= 3\int \frac{\sin t}{\cos t}\,dt - \frac{t}{2}$

$\cos t = u$

$-\sin t\,dt = du$

$= -3\int \frac{1}{u}\,du - \frac{t}{2}$

$= -3\ln|\cos t| - \frac{t}{2} + C$

$= -3\ln\left|\frac{2}{\sqrt{4+(x+1)^2}}\right| - \frac{\arctan\frac{x+1}{2}}{2} + C$

(2018 Bütünleme)

4-) $\int \frac{x^2}{x^3+x^2-x-1}\,dx = \int \frac{x^2}{(x^3-x)+(x^2-1)}\,dx$

$= \int \frac{x^2}{x(x^2-1)+(x^2-1)}\,dx$

$= \int \frac{x^2}{(x-1)(x+1)^2}\,dx$

$$\frac{x^2}{(x-1)(x+1)^2} = \frac{A}{\underset{(x+1)^2}{x-1}} + \frac{B}{\underset{(x-1)(x+1)}{x+1}} + \frac{C}{\underset{(x-1)}{(x+1)^2}} = \frac{Ax^2+2Ax+A+Bx^2-B+Cx-C}{(x-1)(x+1)^2}$$

$A = \frac{1}{4} \quad B = \frac{3}{4} \quad C = -\frac{1}{2}$

$$\int \frac{x^2}{x^3+x^2-x-1}\,dx = \frac{1}{4}\int \frac{1}{x-1}\,dx + \frac{3}{4}\int \frac{1}{x+1}\,dx - \frac{1}{2}\int \frac{1}{(x+1)^2}\,dx$$

$$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2}\cdot\frac{1}{x+1} + C$$

5-) $\int \frac{x^2}{\sqrt{12+4x-x^2}}\,dx = \int \frac{x^2}{\sqrt{-(x^2-4x-12)}}\,dx$

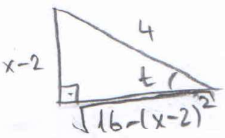
(2018 Final)

$$= \int \frac{x^2}{\sqrt{-((x-2)^2-4^2)}}\,dx$$

$$= \int \frac{x^2}{\sqrt{4^2-(x-2)^2}}\,dx$$

$x-2 = 4\sin t$

$dx = 4\cos t\,dt$



$\sin t = \frac{x-2}{4}$

$t = \arcsin\frac{x-2}{4}$

$$= \int \frac{(2+4\sin t)^2}{\sqrt{4^2(1-\sin^2 t)}}\,4\cos t\,dt$$

$$= \int \frac{4+16\sin^2 t+16\sin t}{\cancel{4.\cos t}}\,\cancel{4\cos t}\,dt$$

$$= 4\int dt + 16\int \sin^2 t\,dt + 16\int \sin t\,dt$$

$$= 4t + 16.\int \frac{1-\cos 2t}{2}\,dt + 16(-\cos t) + C$$

$$= 4t + 8\left(t - \frac{\sin 2t}{2}\right) - 16\cos t + C$$

$$= 12t - 4\sin 2t - 16\cos t + C$$

$$= 12.\arcsin\frac{x-2}{4} - 8.\frac{x-2}{4}.\frac{\sqrt{16-(x-2)^2}}{4} - 16.\frac{\sqrt{16-(x-2)^2}}{4} + C$$

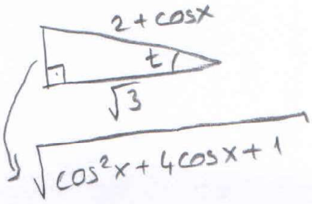
$$= 12\arcsin\frac{x-2}{4} - \frac{1}{2}(x-2).\sqrt{16-(x-2)^2} - 4\sqrt{16-(x-2)^2} + C$$

6) $$\int \frac{\sin x}{\sqrt{\cos^2 x + 4\cos x + 1}}\,dx = \int \frac{\sin x}{\sqrt{(\cos x + 2)^2 - (\sqrt{3})^2}}\,dx$$

$$\boxed{\begin{aligned} 2+\cos x &= \sqrt{3}\sec t \\ -\sin x\,dx &= \sqrt{3}\sec t\tan t\,dt \end{aligned}}$$

$$\sec t = \frac{2+\cos x}{\sqrt{3}} = \frac{1}{\cos t}$$

$$\cos t = \frac{\sqrt{3}}{2+\cos x}$$



$$= \int \frac{-\sqrt{3}\sec t\tan t}{\sqrt{3\sec^2 t - 3}}\,dt$$

$$= \int \frac{-\cancel{\sqrt{3}}\sec t\,\cancel{\tan t}}{\cancel{\sqrt{3}}\,\cancel{\tan t}}\,dt$$

$$= -\int \sec t\,dt$$

$$= -\ln|\sec t + \tan t| + c$$

$$= -\ln\left|\frac{2+\cos x}{\sqrt{3}} + \frac{\sqrt{\cos^2 x + 4\cos x + 1}}{\sqrt{3}}\right| + c$$

7) $$\int \frac{2+\tan x}{1-2\tan x}\,dx = \int \frac{2+u}{1-2u}\,\frac{du}{1+u^2}$$

$$\boxed{\begin{aligned} \tan x &= u \\ (1+\tan^2 x)\,dx &= du \\ dx &= \frac{du}{1+u^2} \end{aligned}}$$

$$\frac{2+u}{(1-2u)(1+u^2)} = \frac{A}{1-2u} + \frac{Bu+C}{1+u^2}$$

$$A = 2,\ B = 1,\ C = 0$$

$$\int \frac{2+\tan x}{1-2\tan x}\,dx = 2\int \frac{1}{1-2u}\,du + \int \frac{u}{1+u^2}\,du$$

$$= 2\,\frac{\ln|1-2u|}{-2} + \frac{1}{2}\ln(1+u^2) + c$$

$$= -\ln|1-2\tan x| + \frac{1}{2}\ln(1+\tan^2 x) + c$$

8) $\int \frac{\cos x}{2+2\sin x-2\cos x}\,dx = \int \frac{\frac{1-t^2}{1+t^2}}{2+2\cdot\frac{2t}{1+t^2}-2\cdot\frac{1-t^2}{1+t^2}}\cdot\frac{2dt}{1+t^2}$

(2018 Yaz Final)

$\tan\frac{x}{2}=t$

$$= \int \frac{1-t^2}{(1+t^2)^2\cdot\left(\frac{\cancel{1}+t^2+2t-\cancel{1}+t^2}{1+t^2}\right)}\,dt$$

$$= \int \frac{1-t^2}{(1+t^2)(2t^2+2t)}\,dt$$

$$= \frac{1}{2}\int \frac{(1-t)\cancel{(1+t)}}{(1+t^2)\,t\cancel{(1+t)}}\,dt$$

$$= \frac{1}{2}\int \frac{1-t}{t(1+t^2)}\,dt$$

$$\frac{1-t}{t(1+t^2)} = \underset{(1+t^2)}{\frac{A}{t}} + \underset{(t)}{\frac{Bt+C}{1+t^2}} = \frac{At^2+A+Bt^2+Ct}{t(1+t^2)}$$

$$\left.\begin{aligned} A+B&=0\\ C&=-1\\ A&=1\end{aligned}\right\} \Rightarrow \boxed{\begin{aligned}A&=1\\ B&=-1\\ C&=-1\end{aligned}}$$

$$\int \frac{\cos x}{2+2\sin x-2\cos x}\,dx = \frac{1}{2}\int\left(\frac{1}{t}-\frac{t+1}{1+t^2}\right)dt$$

$$= \frac{1}{2}\int \frac{1}{t}-\frac{t}{1+t^2}-\frac{1}{1+t^2}\,dt$$

$$= \frac{1}{2}\left[\ln|t| - \frac{1}{2}\cdot\ln(1+t^2) - \arctan t\right]+C$$

$$= \frac{1}{2}\ln\left|\tan\frac{x}{2}\right| - \frac{1}{4}\ln\left(1+\tan^2\frac{x}{2}\right) - \frac{1}{2}\arctan\left(\tan\frac{x}{2}\right)+C$$