

UYGULAMA 11

$$1) \int \frac{\sin x \cos^3 x}{1 + \cos^2 x} dx = \int \frac{-u^3}{1+u^2} du = - \int \left(u - \frac{u}{u^2+1} \right) du$$

$$= -\frac{u^2}{2} + \frac{1}{2} \ln(u^2+1) + C$$

$$= -\frac{\cos^2 x}{2} + \frac{1}{2} \ln(1 + \cos^2 x) + C$$

(2017 Bütünleme)

$\cos x = u$
 $-\sin x dx = du$

$$2) \int \frac{\sqrt{x} - 2 \sqrt[4]{x}}{3 \sqrt[3]{x}} dx = \int \frac{u^6 - 2u^3}{3u^4} \cdot 12u^{\frac{1}{2}} du$$

$$= 4 \int u^{13} - 2u^{10} du$$

$$= 4 \left(\frac{u^{14}}{14} - 2 \frac{u^{11}}{11} \right) + C$$

$$= \frac{2}{7} u^{14} - \frac{8}{11} u^{11} + C$$

$$= \frac{2}{7} x^{7/6} - \frac{8}{11} x^{11/12} + C$$

(2017 Bütünleme)

$$3) \int \sin(\ln x) dx = x \cdot \sin(\ln x) - \int \cos(\ln x) dx$$

$$u = \sin(\ln x) \quad dx = du$$

$$\frac{\cos(\ln x)}{x} dx = du \quad x = u$$

$$\int \cos(\ln x) dx = x \cos(\ln x) + \int \sin(\ln x) dx$$

$$k = \cos(\ln x) \quad dx = dk$$

$$dk = -\frac{\sin(\ln x)}{x} dx \quad x = k$$

$$\int \sin(\ln x) dx = x \sin(\ln x) - \left(x \cos(\ln x) + \int \sin(\ln x) dx \right)$$

$$2 \int \sin(\ln x) dx = x (\sin(\ln x) - \cos(\ln x)) + C$$

$$\int \sin(\ln x) dx = \frac{x (\sin(\ln x) - \cos(\ln x))}{2} + C$$

$$4) \int x^3 \sqrt{1-x^2} dx = - \int \sqrt{t} (1-t) \frac{dt}{2} = -\frac{1}{2} \int (t^{1/2} - t^{3/2}) dt$$

$$1-x^2 = t$$

$$-2x dx = dt$$

$$dx = -\frac{dt}{2x}$$

$$x^3 dx = -x^2 \frac{dt}{2} = -(1-t) \frac{dt}{2}$$

$$= -\frac{1}{2} \left(\frac{t^{3/2}}{3/2} - \frac{t^{5/2}}{5/2} \right) + C$$

$$= -\frac{1}{3} t^{3/2} + \frac{1}{5} t^{5/2} + C$$

$$= -\frac{1}{3} (1-x^2)^{3/2} + \frac{1}{5} (1-x^2)^{5/2} + C$$

$$5) \int \frac{2 dx}{1+2\sin x} = \int \frac{2}{1+2 \cdot \frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2} dt$$

$$\tan \frac{x}{2} = t$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$dx = \frac{2}{1+t^2} dt$$

$$t+2 = \sqrt{3} \sec \theta$$

$$dt = \sqrt{3} \sec \theta \tan \theta d\theta$$

$$= 4 \int \frac{1}{\frac{1+t^2+4t}{1+t^2}} \cdot \frac{1}{1+t^2} dt$$

$$= 4 \int \frac{1}{t^2+4t+1} dt$$

$$= 4 \int \frac{1}{(t+2)^2 - (\sqrt{3})^2} dt$$

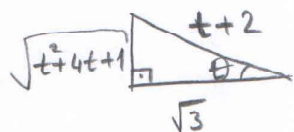
$$= 4 \int \frac{1}{3 \sec^2 \theta - 3} \sqrt{3} \sec \theta \tan \theta d\theta$$

$$= \frac{4}{\sqrt{3}} \int \frac{\sec \theta \tan \theta}{\tan^2 \theta} d\theta = \frac{4}{\sqrt{3}} \int \operatorname{cosec} \theta d\theta$$

$$= -\frac{4}{\sqrt{3}} \ln |\operatorname{cosec} \theta + \cot \theta| + c$$

$$\sec \theta = \frac{t+2}{\sqrt{3}} = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{\sqrt{3}}{t+2}$$



$$= -\frac{4}{\sqrt{3}} \ln \left| \frac{1}{\frac{\sqrt{t^2+4t+1}}{t+2}} + \frac{\sqrt{3}}{\sqrt{t^2+4t+1}} \right| + c$$

$$= -\frac{4}{\sqrt{3}} \ln \left| \frac{t+2+\sqrt{3}}{\sqrt{t^2+4t+1}} \right| + c$$

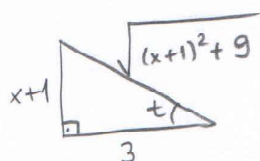
$$= -\frac{4}{\sqrt{3}} \ln \left| \frac{\tan \frac{x}{2} + 2 + \sqrt{3}}{\sqrt{\tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 1}} \right| + c$$

$$6) \int \frac{x+7}{((x+1)^2+9)^2} dx = \int \frac{3\tan t - 1 + 7}{(3^2(\tan^2 t + 1))^2} \cdot 3\sec^2 t dt$$

(2016 Final)

$$\boxed{\begin{aligned} x+1 &= 3\tan t \\ dx &= 3\sec^2 t dt \end{aligned}}$$

$$\tan t = \frac{x+1}{3}$$



$$t = \arctan \frac{x+1}{3}$$

$$= \int \frac{3\tan t + 6}{3^4 (\sec^2 t)^2} 3\sec^2 t dt$$

$$= \int \frac{3(\tan t + 2)}{3^4 \sec^4 t} 3\sec^2 t dt$$

$$= \frac{1}{9} \int \frac{\tan t + 2}{\sec^2 t} dt$$

$$= \frac{1}{9} \int \left(\frac{\sin t}{\cos t} + 2 \right) \cos^2 t dt$$

$$= \frac{1}{9} \left(\int \sin t \cos t dt + \int 2 \cos^2 t dt \right)$$

$$= \frac{1}{9} \frac{\sin^2 t}{2} + \frac{1}{9} \int 2 \cdot \frac{1+\cos 2t}{2} dt$$

$$= \frac{1}{18} \sin^2 t + \frac{1}{9} \left(t + \frac{\sin 2t}{2} \right) + C$$

$$= \frac{1}{18} \sin^2 t + \frac{1}{9} \cdot t + \frac{1}{18} \sin 2t + C$$

$$= \frac{1}{18} \left(\frac{x+1}{\sqrt{(x+1)^2+9}} \right)^2 + \frac{1}{9} \cdot \arctan \frac{x+1}{3} + \frac{1}{18} \cdot 2 \cdot \frac{x+1}{\sqrt{(x+1)^2+9}} \cdot \frac{3}{\sqrt{(x+1)^2+9}} + C$$

$$= \frac{1}{18} \frac{(x+1)^2}{(x+1)^2+9} + \frac{1}{9} \arctan \frac{x+1}{3} + \frac{1}{3} \frac{x+1}{(x+1)^2+9} + C$$

$$7) \int \frac{e^{2x} - e^x}{e^{2x} + 3e^x + 2} dx = \int \frac{e^x(e^x - 1)}{(e^x)^2 + 3e^x + 2} dx$$

$$\boxed{\begin{array}{l} e^x = u \\ e^x dx = du \end{array}}$$

$$= \int \frac{u-1}{u^2+3u+2} du$$

$$\frac{u-1}{u^2+3u+2} = \frac{u-1}{(u+2)(u+1)} = \frac{A}{u+2} + \frac{B}{u+1} \Rightarrow A=3, B=-2$$

$$\begin{aligned} \int \frac{e^{2x} - e^x}{e^{2x} + 3e^x + 2} dx &= \int \left(\frac{3}{u+2} - \frac{2}{u+1} \right) du \\ &= 3 \ln|u+2| - 2 \ln|u+1| + C \\ &= 3 \ln(e^x+2) - 2 \ln(e^x+1) + C \end{aligned}$$

$$8) I_n = \int x^2 (\ln x)^n dx \text{ ise } I_n = \frac{x^3}{3} (\ln x)^n - \frac{n}{3} I_{n-1}$$

olduğunu gösteriniz.

$$\boxed{\begin{array}{l} (\ln x)^n = u \\ n(\ln x)^{n-1} \cdot \frac{1}{x} dx = du \\ x^2 dx = du \\ \frac{x^3}{3} = u \end{array}}$$

$$I_n = \frac{x^3 (\ln x)^n}{3} - \frac{n}{3} \int (\ln x)^{n-1} \frac{1}{x} x^3 dx$$

$$= \frac{x^3 (\ln x)^n}{3} - \frac{n}{3} \int (\ln x)^{n-1} x^2 dx$$

$$= \frac{x^3 (\ln x)^n}{3} - \frac{n}{3} I_{n-1}$$

$$\begin{aligned}
 9) \int \frac{x^5}{(x-1)^3} dx &= \int \frac{(u+1)^5}{u^3} du \\
 x-1 &= u \\
 dx &= du \\
 &= \int \frac{u^5 + 5u^4 + 10u^3 + 10u^2 + 5u + 1}{u^3} du \\
 &= \int u^2 + 5u + 10 + \frac{10}{u} + \frac{5}{u^2} + \frac{1}{u^3} du \\
 &= \frac{u^3}{3} + \frac{5u^2}{2} + 10u + 10 \ln|u| - \frac{5}{u} - \frac{1}{2u^2} + C \\
 &= \frac{(x-1)^3}{3} + \frac{5}{2}(x-1)^2 + 10(x-1) + 10 \ln|x-1| - \frac{5}{x-1} \\
 &\quad - \frac{1}{2(x-1)^2} + C
 \end{aligned}$$

(2018 Bitanleue)

$$\begin{aligned}
 10) \int \frac{\sqrt[3]{3x+5} + 4}{\sqrt[4]{3x+5}} dx &= \int \frac{t^4 + 4}{t^3} \cdot 4t^{1/2} dt \\
 3x+5 &= t^2 \\
 3dx &= 12t^{1/2} dt \\
 dx &= 4t^{1/2} dt \\
 t &= (3x+5)^{1/2} \\
 &= 4 \int t^8 (t^4 + 4) dt \\
 &= 4 \int t^{12} + 4t^8 dt \\
 &= 4 \cdot \frac{t^{13}}{13} + 16 \frac{t^9}{9} + C \\
 &= \frac{4}{13} \cdot (3x+5)^{13/2} + \frac{16}{9} (3x+5)^{9/2} + C
 \end{aligned}$$

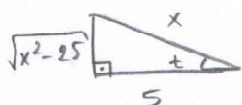
$$11) \int \frac{\sqrt{x^2-25}}{x^3} dx = \int \frac{5\sqrt{\sec^2 t - 1}}{5^3 \cdot \sec^3 t} \cdot 5 \sec t \tan t dt$$

$$x = 5 \sec t$$

$$dx = 5 \sec t \tan t dt$$

$$\sec t = \frac{x}{5} = \frac{1}{\cos t}$$

$$\cos t = \frac{5}{x}$$



$$= \frac{1}{5} \int \frac{\tan t}{\sec^3 t} \sec t \tan t dt$$

$$= \frac{1}{5} \int \frac{\tan^2 t}{\sec^2 t} dt$$

$$= \frac{1}{5} \int \sin^2 t dt$$

$$= \frac{1}{5} \int \frac{1 - \cos 2t}{2} dt$$

$$= \frac{1}{10} \int (1 - \cos 2t) dt$$

$$= \frac{1}{10} \left(t - \frac{\sin 2t}{2} \right) + C = \frac{\operatorname{arccsec} \frac{x}{5}}{10} - \frac{\sqrt{x^2-25}}{2x^2} + C$$

ÖDEV PROBLEMLER

$$1) \int \sqrt{x} \cos \sqrt{x} dx = ?$$

(2018 Yaz Arasınan)

$$2) \int \frac{1 - \sin x}{1 + \cos x} dx = ?$$

$$\int \frac{2^x}{\sqrt{1-4^x}} dx = ?$$

$$\int \frac{\sin 2x}{\sqrt{1 - \sin^4 x}} dx = ?$$

$$\int \frac{3x^3 - 4x^2 + 3x}{x^2 + 1} dx = ?$$

$$3) \int \operatorname{arccot} x dx = ?$$

$$\int (\ln x)^2 dx = ?$$

$$\int \frac{\ln x}{\sqrt{x}} dx = ?$$

$$\int x \ln(x^2) dx = ?$$

$$4) \int \sin 2x \cos 5x dx = ?$$

$$\int \sin^3 x \cos^6 x dx = ?$$

$$5 \rightarrow \int \frac{x+1}{x^3+x^2-6x} dx = ?$$

$$\int \frac{x^2}{(x+2)^3} dx = ?$$

$$6 \rightarrow \int \frac{\sin x}{\cos^2 x + \cos x - 6} dx = ?$$

$$\int \sqrt{x^2+9} dx = ?$$

$$\int \frac{x^4}{x^4-1} dx = ?$$

$$\int \frac{3x^2-x+1}{x^3-1} dx = ?$$

$$\int \frac{dx}{x \cdot \sqrt[3]{x^2+1}}$$

$$\int \frac{dx}{2\sin x + 3\cos x - 5} = ?$$