$$I = \int_{-1}^{4} \left[[\![\tfrac{x}{2}]\!] + \operatorname{sgn}(x-1) \right] dx$$ integralini hesaplayınız.

Çözüm:

$$I = \int_{-1}^{4} [\![\tfrac{x}{2}]\!]\, dx + \int_{-1}^{4} \operatorname{sgn}(x-1)\, dx$$

$$\int_{-1}^{4} [\![\tfrac{x}{2}]\!]\, dx = \int_{-1}^{0} (-1)\, dx + \int_{0}^{2} 0\, dx + \int_{2}^{4} dx = -x\Big|_{-1}^{0} + 0 + x\Big|_{2}^{4} = -1+2 = 1$$

$-1 < x < 4 \Rightarrow -\frac{1}{2} < \frac{x}{2} < 2 \Rightarrow [\![\frac{x}{2}]\!] = -1,\ [\![\frac{x}{2}]\!] = 0,\ [\![\frac{x}{2}]\!] = 1$ olabilir

$$[\![\tfrac{x}{2}]\!] = -1 \Rightarrow -1 \leq \tfrac{x}{2} < 0 \Rightarrow -2 \leq x < 0$$

$$[\![\tfrac{x}{2}]\!] = 0 \Rightarrow 0 \leq \tfrac{x}{2} < 1 \Rightarrow 0 \leq x < 2$$

$$[\![\tfrac{x}{2}]\!] = 1 \Rightarrow 1 \leq \tfrac{x}{2} < 2 \Rightarrow 2 \leq x < 4$$

$$\int_{-1}^{4} \operatorname{sgn}(x-1)\, dx = \int_{-1}^{1} \operatorname{sgn}(x-1)\, dx + \int_{1}^{4} \operatorname{sgn}(x-1)\, dx = \int_{-1}^{1} -dx + \int_{1}^{4} dx$$

$x - 1 = 0 \Rightarrow x = 1$

	-1	1	4
$x-1$	$-$		$+$
$\operatorname{sgn}(x-1)$	-1		$+1$

$$= -x\Big|_{-1}^{1} + x\Big|_{1}^{4}$$

$$= -(1-(-1)) + 3$$

$$= 1$$

$$I = 1 + 1 = 2$$

(2016 Final)

7)

$$\int_{-2}^{1} x^2|2x-1|\,dx = \int_{-2}^{1/2} x^2|2x-1|\,dx + \int_{1/2}^{1} x^2|2x-1|\,dy$$

$2x-1=0$

$x=\frac{1}{2}$

$$\begin{array}{c|c} \multicolumn{2}{c}{1/2} \\ \hline - & + \end{array}$$

$$= \int_{-2}^{1/2} x^2(1-2x)\,dx + \int_{1/2}^{1} x^2(2x-1)\,dx$$

$$= \int_{-2}^{1/2} x^2-2x^3\,dx + \int_{1/2}^{1} 2x^3-x^2\,dx$$

$$= \left(\frac{x^3}{3}-\frac{x^4}{2}\right)\Bigg|_{-2}^{1/2} + \left(\frac{x^4}{2}-\frac{x^3}{3}\right)\Bigg|_{1/2}^{1}$$

$$= \frac{1}{24}-\frac{1}{32}+\frac{8}{3}+8+\frac{1}{2}-\frac{1}{3}-\frac{1}{32}+\frac{1}{24}$$

$$= \frac{521}{48}$$

$$\int \frac{\sin x.\cos^3 x}{1+\cos^2 x}\,dx = -\int \frac{u^3}{1+u^2}\,du = -\int u-\frac{u}{u^2+1}\,du$$

$\cos x = u$

$-\sin x\,dx = du$

$\sin x\,dx = -du$

$$\begin{array}{r|l} u^3 & u^2+1 \\ \cline{2-2} -\ u^3+u & u \\ \hline -u & \end{array}$$

$$= -\frac{u^2}{2} + \int \frac{u}{u^2+1}\,du$$

$$= -\frac{u^2}{2} + \frac{\ln(u^2+1)}{2} + C$$

$$= -\frac{\cos^2 x}{2} + \frac{\ln(1+\cos^2 x)}{2} + C$$

(2017 Bütünleme)

$f: [2,5] \to \mathbb{R}$, $f(x) = x^2 + \frac{1}{x}$ fonksiyonu verilsin. $[2,5]$ aralığını 3 eşit parçaya bölerek oluşturulan P bölüntüsü için $A(f,P)$ alt toplamını bulunuz.

$f'(x) = 2x - \frac{1}{x^2}$, $x \in [2,5]$ olduğundan f artandır.

$P = \{2, 3, 4, 5\}$

$[2,3]$, $[3,4]$, $[4,5]$

$\Delta x_1 = \Delta x_2 = \Delta x_3 = 1$

$m_1 = \inf\{f(x) : x \in [2,3]\} = f(2) = \frac{9}{2}$

$m_2 = \inf\{f(x) : x \in [3,4]\} = f(3) = \frac{28}{3}$

$m_3 = \inf\{f(x) : x \in [4,5]\} = f(4) = \frac{65}{4}$

$$A(f,P) = 1 \cdot \underset{(6)}{\frac{9}{2}} + 1 \cdot \underset{(4)}{\frac{28}{3}} + 1 \cdot \underset{(3)}{\frac{65}{4}}$$

$$= \frac{54 + 112 + 195}{12}$$

$$= \frac{361}{12}$$

(2016 Bütünleme)

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+n}}+\frac{1}{\sqrt{n^2+2n}}+\dots+\frac{1}{\sqrt{n^2+(n-1)n}}+\frac{1}{\sqrt{2n^2}}\right]$$

limitini hesaplayınız.

$$\lim_{n\to\infty}\frac{1}{\sqrt{n^2+n}}+\frac{1}{\sqrt{n^2+2n}}+\dots+\frac{1}{\sqrt{n^2+n.n}}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{n^2+kn}}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n.\sqrt{1+\frac{k}{n}}}$$

$$f(x)=\frac{1}{\sqrt{1+x}}$$

$$=\lim_{n\to\infty}\frac{1}{n}.\sum_{k=1}^{n}\frac{1}{\sqrt{1+\frac{k}{n}}}$$

$$\frac{b-a}{n}=\frac{1}{n}$$

$$a+\frac{b-a}{n}.k=\frac{k}{n}$$

$$=\int_0^1\frac{1}{\sqrt{1+x}}\,dx$$

$a=0,\ b=1$

$$=2(1+x)^{1/2}\Big|_0^1=2\sqrt{2}-2 \; //$$

(2016 Bütünleme)