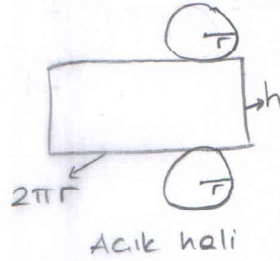
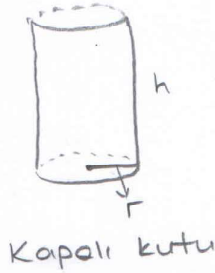


UYGULAMA 9

1) 100 cm^3 hacminde daire tabanlı dik silindirin biçiminde bir kapalı kutu yapılmak isteniyor. Yüzey alanının en küçük olması için bu silindirin h yüksekliği ve r taban yarıçapı ne olmalıdır?

Gözüm:



$$V = \pi \cdot r^2 \cdot h = 100 \Rightarrow h = \frac{100}{\pi r^2}$$

Yüzey Alanı: $A = 2\pi r \cdot h + 2\pi r^2$

$$A(r) = 2\pi r \cdot \frac{100}{\pi r^2} + 2\pi r^2$$

$$A(r) = \frac{200}{r} + 2\pi r^2$$

$$A'(r) = -\frac{200}{r^2} + 4\pi r$$

$$A'(r) = 0 \Leftrightarrow r^3 = \frac{50}{\pi}$$

$$\Leftrightarrow r = \sqrt[3]{\frac{50}{\pi}}$$

$$A''(r) = 4\pi + \frac{400}{r^3}$$

$$A''\left(\sqrt[3]{\frac{50}{\pi}}\right) = 12\pi > 0$$

$$r = \sqrt[3]{\frac{50}{\pi}} \text{ min. yapan nokta}$$

$$h = \frac{100}{\pi \cdot \left(\frac{50}{\pi}\right)^{2/3}}$$

$$2) \int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$$

$$= \arcsin(e^x) + C$$

$$e^x = u$$

$$e^x dx = du$$

(2018 Arasınar)

$$\begin{aligned}
 3) \int \sin^{-6} x \cos^7 x \, dx &= \int \sin^{-6} x \cos^6 x \cos x \, dx \\
 &= \int \sin^{-6} x (\cos^2 x)^3 \cos x \, dx \\
 &= \int \sin^{-6} x (1 - \sin^2 x)^3 \cos x \, dx \\
 &= \int u^{-6} (1 - u^2)^3 \, du \\
 &= \int u^{-6} (1 - 3u^2 + 3u^4 - u^6) \, du \\
 &= \int u^{-6} - 3u^{-4} + 3u^{-2} - u^6 \, du \\
 &= \frac{u^{-5}}{-5} - 3 \frac{u^{-3}}{-3} + 3 \frac{u^{-1}}{-1} - u^7 + C \\
 &= -\frac{1}{5} (\sin x)^{-5} + (\sin x)^{-3} - 3(\sin x)^{-1} - \sin x + C
 \end{aligned}$$

$$\begin{aligned}
 u &= \sin x \\
 du &= \cos x \, dx
 \end{aligned}$$

(2018 Arasınar)

$$\begin{aligned}
 4) \int \operatorname{cosec}^6 x \, dx &= \int \operatorname{cosec}^4 x \operatorname{cosec}^2 x \, dx \\
 &= \int (\operatorname{cosec}^2 x)^2 (\operatorname{cosec}^2 x) \, dx \\
 &= \int (1 + \cot^2 x)^2 (\operatorname{cosec}^2 x) \, dx \\
 &= - \int (1 + u^2)^2 \, du \\
 &= - \int 1 + 2u^2 + u^4 \, du \\
 &= - \left(u + \frac{2u^3}{3} + \frac{u^5}{5} \right) + C \\
 &= -\cot x - \frac{2}{3} \cot^3 x - \frac{1}{5} \cot^5 x + C
 \end{aligned}$$

$$\begin{aligned}
 \cot x &= u \\
 -\operatorname{cosec}^2 x \, dx &= du
 \end{aligned}$$

$$5) \int \sqrt{1+\sin x} \, dx = \int \sqrt{1+\sin x} \frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}} \, dx$$

$$= \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}} \, dx$$

$$= \int \frac{\cos x}{\sqrt{1-\sin x}} \, dx$$

$$\begin{aligned} 1-\sin x &= u \\ -\cos x \, dx &= du \end{aligned}$$

$$= - \int \frac{1}{\sqrt{u}} \, du$$

$$= - \int u^{-1/2} \, du$$

$$= - \frac{u^{1/2}}{1/2} + C = -2\sqrt{1-\sin x} + C$$

$$6) \int \frac{1}{1+e^x} \, dx = \int 1 - \frac{e^x}{1+e^x} \, dx = \int dx - \int \frac{e^x}{1+e^x} \, dx$$

$$\begin{aligned} 1+e^x &= u \\ e^x \, dx &= du \end{aligned}$$

$$= x - \int \frac{du}{u}$$

$$= x - \ln|u| + C$$

$$= x - \ln(1+e^x) + C$$

$$7) \int x \cdot (\arctan x)^2 dx = \frac{x^2 (\arctan x)^2}{2} - \int \frac{x^2}{1+x^2} \arctan x dx$$

$$\boxed{\begin{aligned} (\arctan x)^2 &= u & x dx &= du \\ 2(\arctan x) \cdot \frac{1}{1+x^2} dx &= du \\ \frac{x^2}{2} &= v \end{aligned}}$$

$$= \frac{x^2 (\arctan x)^2}{2} - \int \left(1 - \frac{1}{1+x^2}\right) \arctan x dx$$

$$= \frac{x^2 (\arctan x)^2}{2} - \int \arctan x dx + \int \frac{1}{1+x^2} \arctan x dx$$

$$= \frac{x^2 (\arctan x)^2}{2} - x \arctan x + \frac{1}{2} \ln(1+x^2) + \frac{(\arctan x)^2}{2} + C$$

$$\int \frac{\arctan x}{1+x^2} = \int u du = \frac{u^2}{2} + C = \frac{(\arctan x)^2}{2} + C_1$$

$$\boxed{\begin{aligned} \arctan x &= u \\ \frac{1}{1+x^2} dx &= du \end{aligned}}$$

$$\int \arctan x dx = x \cdot \arctan x - \int \frac{x}{1+x^2} dx = x \arctan x - \frac{1}{2} \ln(1+x^2) + C_2$$

$$\boxed{\begin{aligned} \arctan x &= u & dx &= du \\ \frac{1}{1+x^2} dx &= du & x &= v \end{aligned}}$$

$$8) \int (\ln(\cosh x)) \cdot \tanh x dx = \int (\ln(\cosh x)) \frac{\sinh x}{\cosh x} dx$$

$$\boxed{\begin{aligned} \cosh x &= u \\ \sinh x dx &= du \end{aligned}}$$

$$\ln u = v$$

$$\frac{1}{u} du = dv$$

$$= \int \ln u \cdot \frac{1}{u} du$$

$$= \int v dv$$

$$= \frac{v^2}{2} + C$$

$$= \frac{(\ln u)^2}{2} + C$$

$$= \frac{(\ln(\cosh x))^2}{2} + C$$

$$9) \int \frac{\sin \sqrt{\theta}}{\sqrt{\theta} \cdot \cos^3 \sqrt{\theta}} d\theta = \int \frac{\sin \sqrt{\theta}}{\sqrt{\theta}} \cdot \frac{1}{(\cos \sqrt{\theta})^{3/2}} d\theta$$

$$\begin{aligned} \cos \sqrt{\theta} &= u \\ (-\sin \sqrt{\theta}) \cdot \frac{1}{2\sqrt{\theta}} d\theta &= du \\ \frac{\sin \sqrt{\theta}}{\sqrt{\theta}} d\theta &= -2 du \end{aligned}$$

$$= -2 \int \frac{1}{u^{3/2}} du$$

$$= -2 \int u^{-3/2} du$$

$$= -2 \frac{u^{-1/2}}{-1/2} + C$$

$$= \frac{4}{\sqrt{u}} + C = \frac{4}{\sqrt{\cos \sqrt{\theta}}} + C$$

(2016 Arasınar)

$$10) \int \sqrt{x} \cos \sqrt{x} dx = \int (t \cos t) \cdot 2t dt$$

$$\begin{aligned} t^2 &= x \\ 2t dt &= dx \\ t &= \sqrt{x} \end{aligned}$$

$$= 2 \int t^2 \cos t dt$$

$$= 2 \left(t^2 \sin t - 2 \int t \sin t dt \right)$$

$$= 2t^2 \sin t - 4(-t \cos t + \sin t) + C$$

$$= 2t^2 \sin t + 4t \cos t - 4 \sin t + C$$

$$= 2x \sin \sqrt{x} + 4\sqrt{x} \cos \sqrt{x} - 4 \sin \sqrt{x} + C$$

$$\int t \sin t dt = -t \cos t + \int \cos t dt = -t \cos t + \sin t + C,$$

$$\begin{aligned} t &= k & \sin t dt &= dr \\ dt &= dk & -\cos t &= r \end{aligned}$$

$$11) \int \frac{\ln(2x)}{x \ln(4x)} dx = \int \frac{\ln(2x)}{x (\ln 2 + \ln(2x))} dx = \int \frac{u - \ln 2}{u} du$$

$$\ln 4x = \ln(2 \cdot 2x) = \ln 2 + \ln 2x$$

$$\ln 2 + \ln 2x = u$$

$$\frac{1}{2x} \cdot 2 dx = du$$

$$\frac{dx}{x} = du$$

$$= \int 1 du - \ln 2 \int \frac{1}{u} du$$

$$= u - \ln 2 \ln|u| + C$$

$$= \ln 2 + \ln 2x - (\ln 2)(\ln(\ln 2 + \ln 2x)) + C$$

$$= \ln 4x - \ln 2 \ln|\ln 4x| + C$$

(2018 Yaz Arasınan)