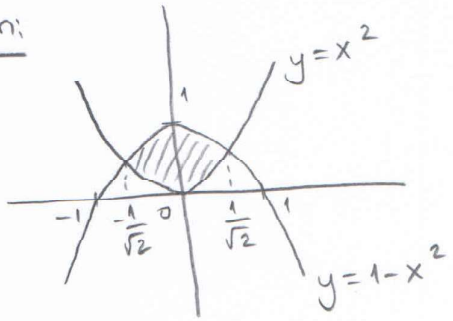
Uygulama 14

1) $y=1-x^2$ ve $y=x^2$ eğrileri ile sınırlı bölgenin alanını bulunuz.

Çözüm:



$$1-x^2=x^2$$

$$x=\mp\frac{1}{\sqrt{2}}$$

$$A=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left((1-x^2)-x^2\right)dx=\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\underbrace{(1-2x^2)}_{\text{çift fonk.}}dx=2\int_0^{\frac{1}{\sqrt{2}}}(1-2x^2)\,dx$$

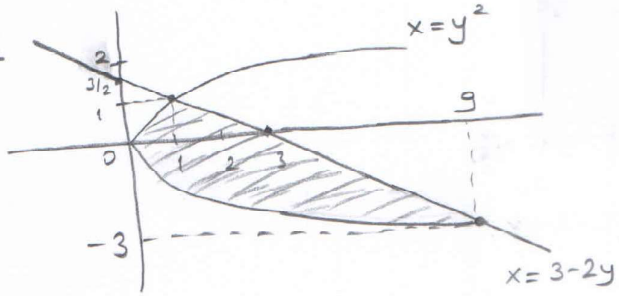
$$=2\left(x-2\frac{x^3}{3}\right)\Big|_0^{\frac{1}{\sqrt{2}}}$$

$$=2\left(\frac{1}{\sqrt{2}}-\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}\sqrt{2}}\right)=\frac{2\sqrt{2}}{3}$$

(2016 Bütünleme)

2) $x=y^2$ parabolü ve $x=3-2y$ doğrusu ile sınırlı bölgenin alanını bulunuz.

Çözüm:



$$y^2=3-2y$$

$$y^2+2y-3=0$$

$$(y+3)(y-1)=0$$

$$y=-3\ ,\quad y=1$$

$$x=9\qquad x=1$$

$$A=\int_{-3}^{1}\left((3-2y)-y^2\right)dy=\left(3y-y^2-\frac{y^3}{3}\right)\Big|_{-3}^{1}$$

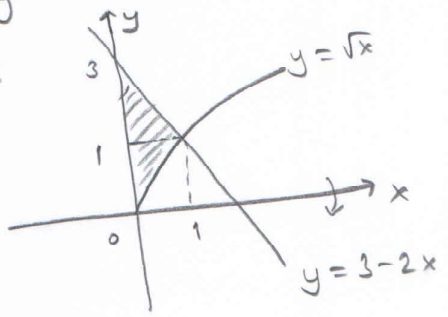
$$=\left(3-1-\frac{1}{3}\right)-\left(-9-\cancel{9}+\cancel{9}\right)$$

$$=\frac{5}{3}+9=\frac{32}{3}$$

(2018 Yaz Final)

3) $y=\sqrt{x}$, $y=3-2x$, $x=0$ ile sınırlı bölgenin x ekseni etrafında dönmesiyle oluşan hacmi bulunuz

Çözüm:



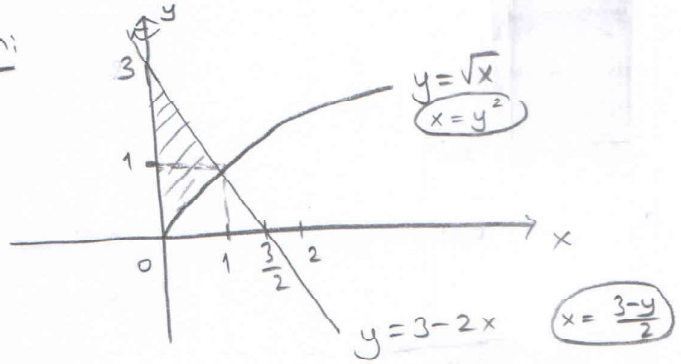
$$V = \pi \int_0^1 \left((3-2x)^2 - (\sqrt{x})^2 \right) dx = \pi \int_0^1 (9+4x^2-12x-x)\, dx$$

$$= \pi \int_0^1 (4x^2-13x+9)\, dx = \pi \cdot \left(\frac{4x^3}{3} - \frac{13x^2}{2} + 9x \right) \Big|_0^1$$

$$= \frac{23}{6}\pi$$

4) $y=\sqrt{x}$, $y=3-2x$, $x=0$ ile sınırlı bölgenin y ekseni etrafında dönmesi ile oluşan hacmi bulunuz.

Çözüm:

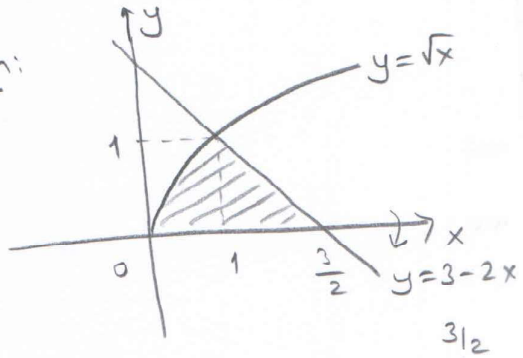


$$V = \pi \int_0^1 (y^2)^2\, dy + \pi \int_1^3 \left(\frac{3-y}{2}\right)^2 dy = \pi \int_0^1 y^4\, dy + \frac{\pi}{4} \int_1^3 (9+y^2-6y)\, dy$$

$$= \pi \cdot \frac{y^5}{5} \Big|_0^1 + \frac{\pi}{4} \left(9y + \frac{y^3}{3} - 3y^2 \right) \Big|_1^3 = \frac{13\pi}{15}$$

5) $y=\sqrt{x}$, $y=3-2x$, $y=0$ ile sınırlı bölgenin x ekseni etrafında dönmesi ile oluşan hacmi bulunuz.

Çözüm:



$$V=\pi\int_0^1(\sqrt{x})^2\,dx+\pi\int_1^{3/2}(3-2x)^2\,dx=\pi\int_0^1 x\,dx+\pi\int_1^{3/2}(9+4x^2-12x)\,dx$$

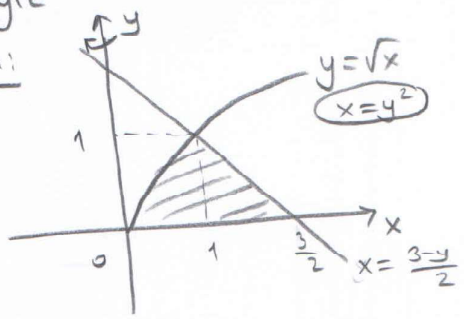
$$=\pi\left(\frac{x^2}{2}\right)\Big|_0^1+\pi\left(9x+4\frac{x^3}{3}-6x^2\right)\Big|_1^{3/2}$$

$$=\frac{\pi}{2}+\frac{\pi}{6}=\frac{4\pi}{6}=\frac{2\pi}{3}$$

(3)

6) $y=\sqrt{x}$, $y=3-2x$, $y=0$ ile sınırlı bölgenin y ekseni etrafında dönmesiyle oluşan hacmi bulunuz.

Çözüm:



$$V=\pi\int_0^1\left(\left(\frac{3-y}{2}\right)^2-(y^2)^2\right)dy=\pi\int_0^1\left(\frac{9}{4}+\frac{y^2}{4}-3y-y^4\right)dy$$

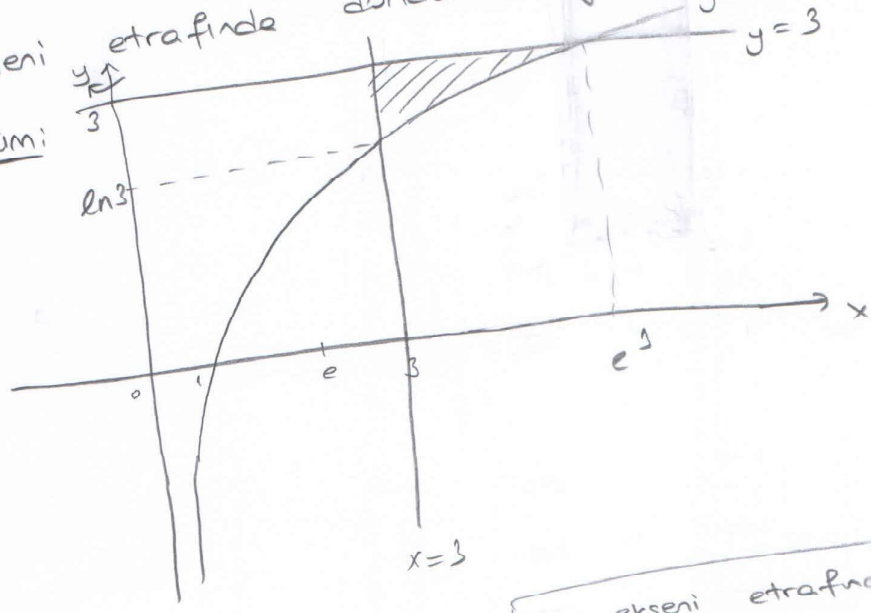
$$=\pi\left(\frac{9}{4}y+\frac{y^3}{12}-\frac{3y^2}{2}-\frac{y^5}{5}\right)\Big|_0^1$$

$$=\pi\left(\frac{9}{4}+\frac{1}{12}-\frac{3}{2}-\frac{1}{5}\right)=\frac{19}{30}\pi$$

7) $y = \ln x$ eğrisi, $x = 3$ ve $y = 3$ doğruları ile sınırlı bölgenin y ekseni etrafında döndürülmesiyle oluşan hacmi bulunuz.

$x = e^y$

Çözüm:



$$V = \pi \int_{\ln 3}^{3} \left((e^y)^2 - 3^2 \right) dy$$

$$= \pi \int_{\ln 3}^{3} \left(e^{2y} - 9 \right) dy$$

$$= \pi \left(\frac{e^{2y}}{2} - 9y \right)_{\ln 3}^{3}$$

$$= \left(\frac{e^6}{2} + 9\ln 3 - \frac{63}{2} \right) \pi$$

x ekseni etrafında döndürülseydi

$$V = \pi \int_{3}^{e^3} \left(3^2 - (\ln x)^2 \right) dx$$