

Chapter 9: Phase Diagrams

ISSUES TO ADDRESS...

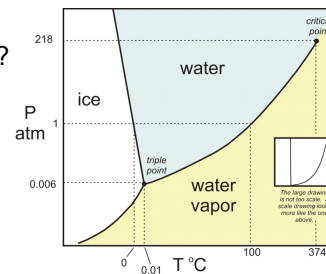
- When we combine two elements...
what is the resulting equilibrium state?
- In particular, if we specify...
 - the composition (e.g., wt% Cu - wt% Ni), and
 - the temperature (T)

then...

How many phases form?

What is the composition of each phase?

What is the amount of each phase?

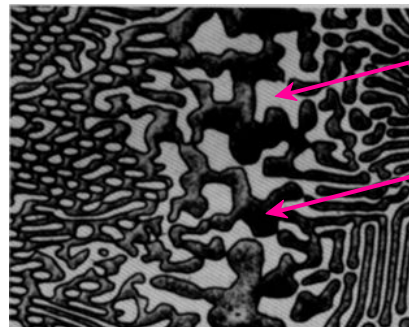


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Components and Phases

- **Components:**
The elements or compounds which are present in the alloy (e.g., Al and Cu)
- **Phases:**
The physically and chemically distinct material regions that form (e.g., α and β).

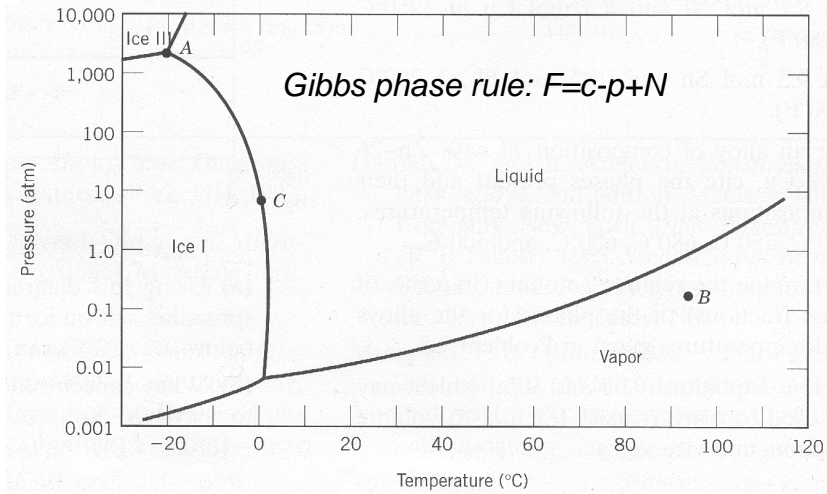
Aluminum-Copper Alloy



Adapted from chapter-opening photograph, Chapter 9, Callister, *Materials Science & Engineering: An Introduction*, 3e.

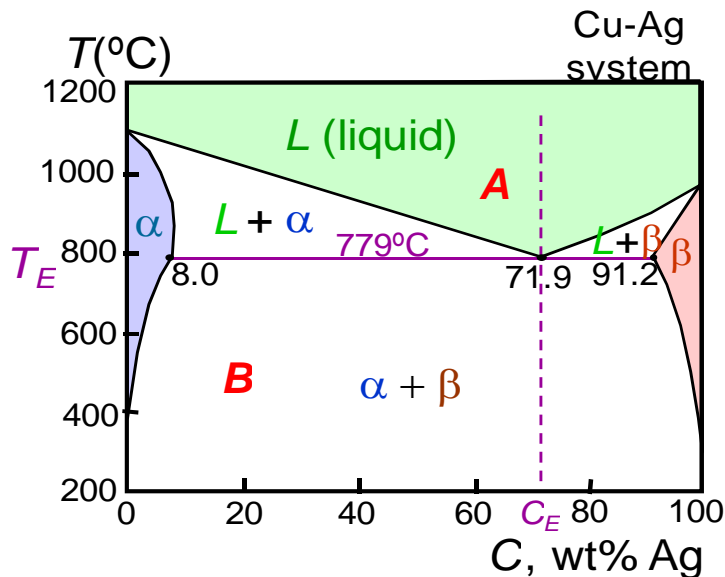
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In the following Figure is shown the pressure–temperature phase diagram for H₂O. Apply the Gibbs phase rule at points A, B, and C; that is, specify the number of degrees of freedom at each of the points—that is, the number of externally controllable variables that need be specified to completely define the system.

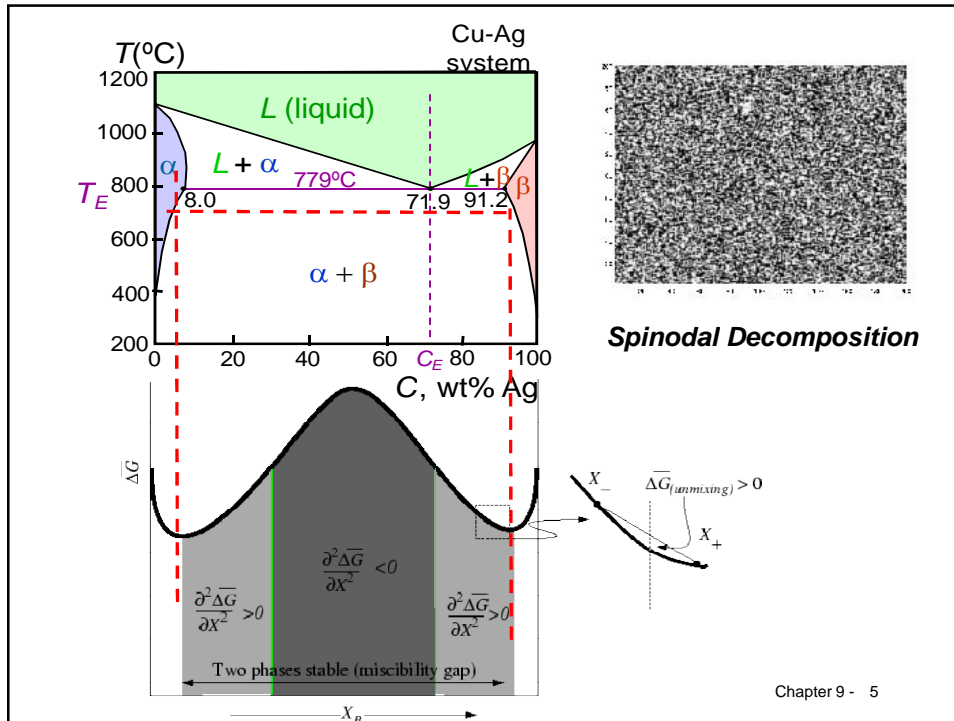


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Apply the Gibbs phase rule at points A and B, and specify the number of degrees of freedom.



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Phase Equilibria: Solubility Limit

Adapted from Fig. 9.1,
Callister & Rethwisch 8e.

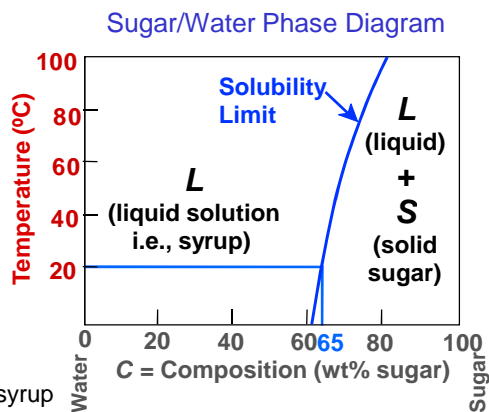
- Solubility Limit:**
Maximum concentration for which only a single phase solution exists.

Question: What is the solubility limit for sugar in water at 20°C ?

Answer: **65 wt% sugar.**

At 20°C , if $C < 65 \text{ wt\%}$ sugar: syrup

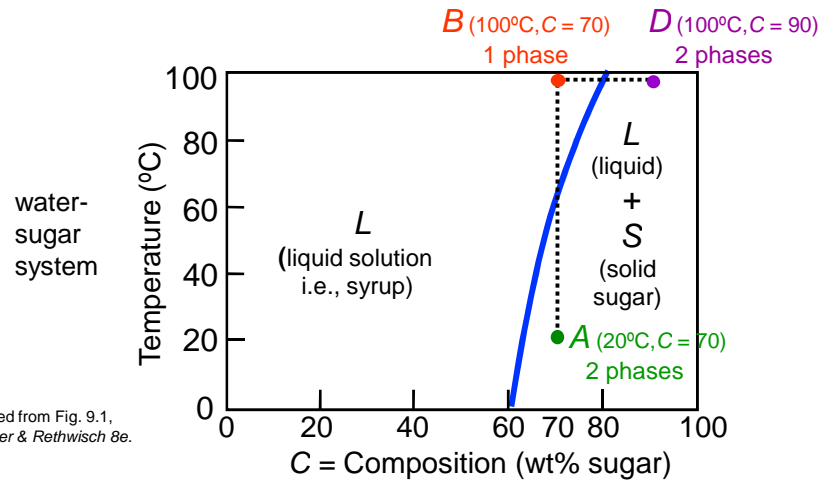
At 20°C , if $C > 65 \text{ wt\%}$ sugar:
syrup + sugar



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Effect of Temperature & Composition

- Altering T can change # of phases: path A to B .
- Altering C can change # of phases: path B to D .



Criteria for Solid Solubility

Simple system (e.g., Ni-Cu solution)

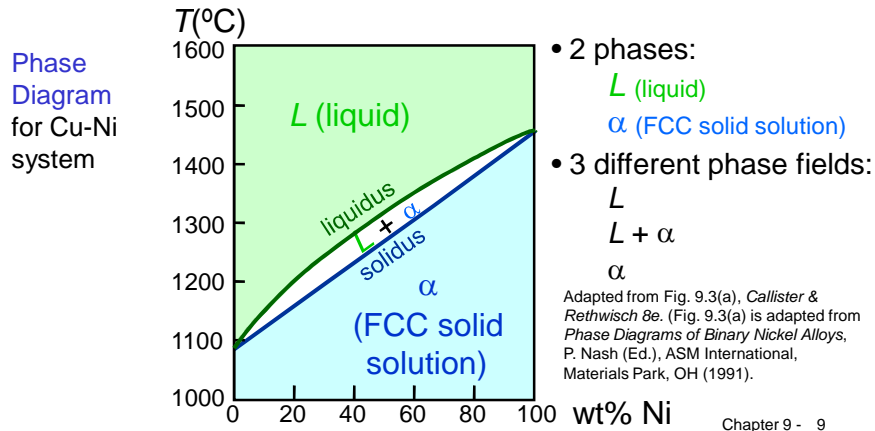
	Crystal Structure	electroneg	r (nm)
Ni	FCC	1.9	0.1246
Cu	FCC	1.8	0.1278

- Both have the same crystal structure (FCC) and have similar electronegativities and atomic radii ([W. Hume – Rothery rules](#)) suggesting high mutual solubility.
- Ni and Cu are totally soluble in one another for all proportions.

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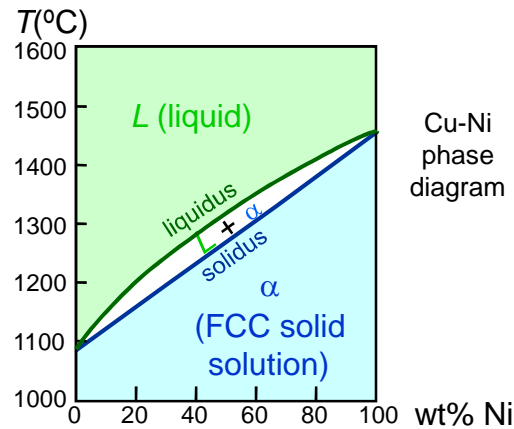
Phase Diagrams

- Indicate phases as a function of T , C , and P .
- For this course:
 - binary systems: just 2 components.
 - independent variables: T and C ($P = 1$ atm is almost always used).



Isomorphous Binary Phase Diagram

- Phase diagram: Cu-Ni system.
- System is:
 - binary
i.e., 2 components: Cu and Ni.
 - isomorphous
i.e., complete solubility of one component in another; α phase field extends from 0 to 100 wt% Ni.



Adapted from Fig. 9.3(a), Callister & Rethwisch 8e. (Fig. 9.3(a) is adapted from Phase Diagrams of Binary Nickel Alloys, P. Nash (Ed.), ASM International, Materials Park, OH (1991).)

Phase Diagrams: Determination of phase(s) present

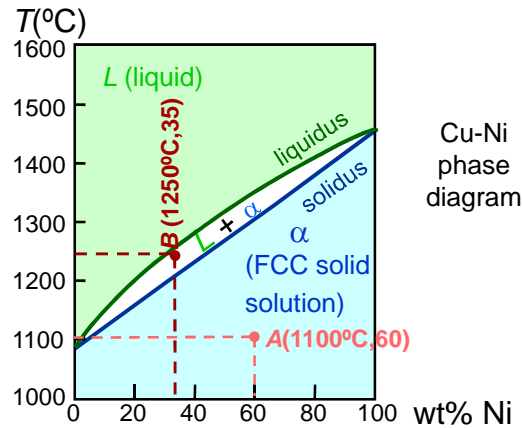
- Rule 1: If we know T and C_0 , then we know:
 - which phase(s) is (are) present.

- Examples:

$A(1100^\circ\text{C}, 60 \text{ wt\% Ni})$:
1 phase: α

$B(1250^\circ\text{C}, 35 \text{ wt\% Ni})$:
2 phases: $L + \alpha$

Adapted from Fig. 9.3(a), Callister & Rethwisch 8e. (Fig. 9.3(a) is adapted from Phase Diagrams of Binary Nickel Alloys, P. Nash (Ed.), ASM International, Materials Park, OH (1991).



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Phase Diagrams: Determination of phase compositions

- Rule 2: If we know T and C_0 , then we can determine:
 - the composition of each phase.

- Examples:

Consider $C_0 = 35 \text{ wt\% Ni}$

At $T_A = 1320^\circ\text{C}$:

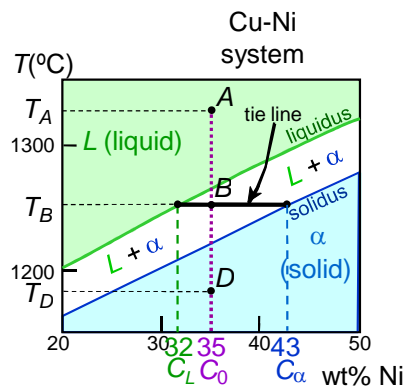
Only Liquid (L) present
 $C_L = C_0 (= 35 \text{ wt\% Ni})$

At $T_D = 1190^\circ\text{C}$:

Only Solid (α) present
 $C_\alpha = C_0 (= 35 \text{ wt\% Ni})$

At $T_B = 1250^\circ\text{C}$:

Both α and L present
 $C_L = C_{\text{liquidus}} (= 32 \text{ wt\% Ni})$
 $C_\alpha = C_{\text{solidus}} (= 43 \text{ wt\% Ni})$



Adapted from Fig. 9.3(a), Callister & Rethwisch 8e. (Fig. 9.3(a) is adapted from Phase Diagrams of Binary Nickel Alloys, P. Nash (Ed.), ASM International, Materials Park, OH (1991).

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Phase Diagrams: Determination of phase weight fractions

- Rule 3: If we know T and C_0 , then can determine:
 - the weight fraction of each phase.
- Examples:
 - Consider $C_0 = 35 \text{ wt\% Ni}$
 - At T_A : Only Liquid (L) present
 - $W_L = 1.00, W_\alpha = 0$
 - At T_D : Only Solid (α) present
 - $W_L = 0, W_\alpha = 1.00$
 - At T_B : Both α and L present

$$W_L = \frac{S}{R+S} = \frac{43-35}{43-32} = 0.73$$

$$W_\alpha = \frac{R}{R+S} = 0.27$$

Adapted from Fig. 9.3(a), Callister & Rethwisch 8e. (Fig. 9.3(a) is adapted from Phase Diagrams of Binary Nickel Alloys, P. Nash (Ed.), ASM International, Materials Park, OH (1991).)

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The Lever Rule

- Tie line – connects the phases in equilibrium with each other – also sometimes called an **isotherm**

Adapted from Fig. 9.3(b), Callister & Rethwisch 8e.

What fraction of each phase?
Think of the tie line as a lever (teeter-totter)

$M_L \times R = M_\alpha \times S$

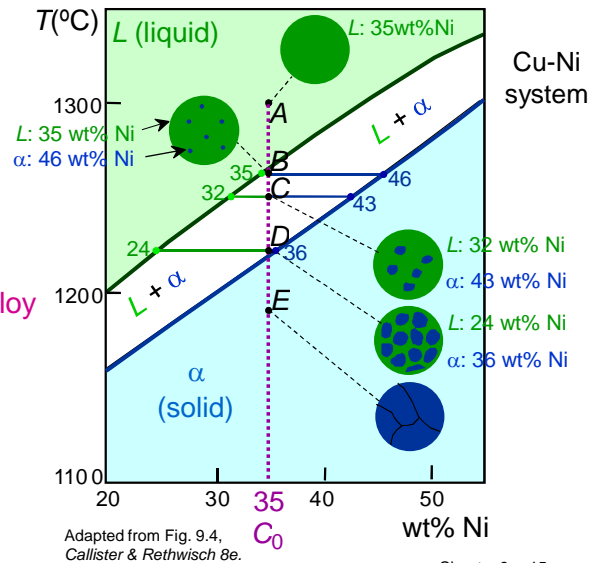
$$W_L = \frac{M_L}{M_L + M_\alpha} = \frac{S}{R+S} = \frac{C_\alpha - C_0}{C_\alpha - C_L}$$

$$W_\alpha = \frac{M_\alpha}{M_L + M_\alpha} = \frac{R}{R+S} = \frac{C_0 - C_L}{C_\alpha - C_L}$$

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Ex: Cooling of a Cu-Ni Alloy

- Phase diagram: Cu-Ni system.
- Consider microstructural changes that accompany the cooling of a $C_0 = 35 \text{ wt\% Ni}$ alloy



2 Component System – no or limited solubility

Invariant Reactions in Binary Alloys

Invariant reaction	Phase reaction	Phase diagram
Eutectic	$L \rightarrow \alpha(s) + \beta(s)$ cooling	
Peritectic	$\alpha(s) + L \rightarrow \beta(s)$ cooling	
Eutectoid	$\gamma(s) \rightarrow \alpha(s) + \beta(s)$ cooling	
Peritectoid	$\alpha(s) + \gamma(s) \rightarrow \beta(s)$ cooling	

Binary-Eutectic Systems

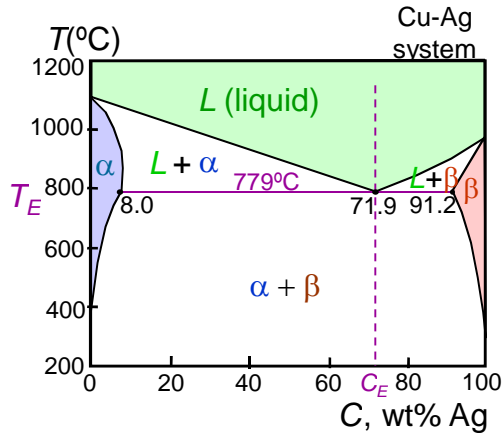
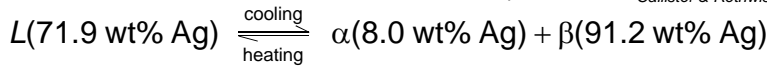
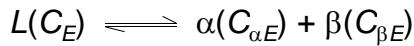
2 components

has a special composition with a min. melting T .

Ex.: Cu-Ag system

- 3 single phase regions (L, α, β)
- Limited solubility:
 - α : mostly Cu
 - β : mostly Ag
- T_E : No liquid below T_E
- C_E : Composition at temperature T_E

• Eutectic reaction



Adapted from Fig. 9.7, Callister & Rethwisch 8e.

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EX 1: Pb-Sn Eutectic System

- For a 40 wt% Sn-60 wt% Pb alloy at 150°C, determine:
 - the phases present
 - the phase compositions
 - the relative amount of each phase

Answer: $\alpha + \beta$

Answer: $C_\alpha = 11 \text{ wt\% Sn}$
 $C_\beta = 99 \text{ wt\% Sn}$

-- the relative amount of each phase

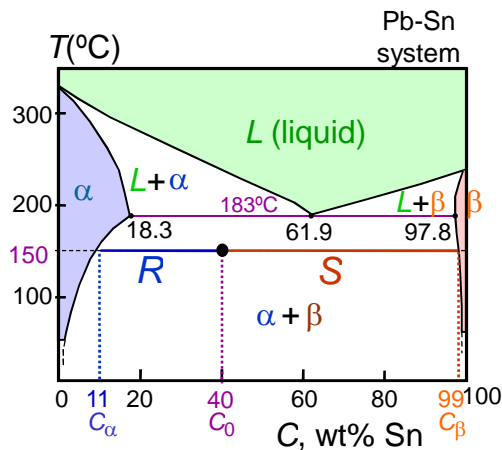
Answer:

$$W_\alpha = \frac{S}{R+S} = \frac{C_\beta - C_0}{C_\beta - C_\alpha}$$

$$= \frac{99 - 40}{99 - 11} = \frac{59}{88} = 0.67$$

$$W_\beta = \frac{R}{R+S} = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha}$$

$$= \frac{40 - 11}{99 - 11} = \frac{29}{88} = 0.33$$



Adapted from Fig. 9.8, Callister & Rethwisch 8e.

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EX 2: Pb-Sn Eutectic System

- For a 40 wt% Sn-60 wt% Pb alloy at 220°C, determine:
 - the phases present:

Answer: $\alpha + L$

- the phase compositions

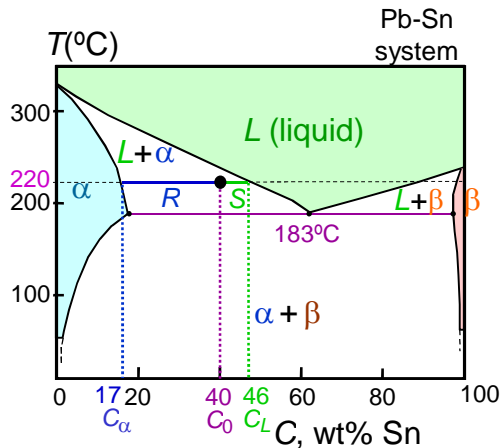
Answer: $C_\alpha = 17 \text{ wt\% Sn}$
 $C_L = 46 \text{ wt\% Sn}$

- the relative amount of each phase

Answer:

$$W_\alpha = \frac{C_L - C_0}{C_L - C_\alpha} = \frac{46 - 40}{46 - 17} = \frac{6}{29} = 0.21$$

$$W_L = \frac{C_0 - C_\alpha}{C_L - C_\alpha} = \frac{23}{29} = 0.79$$

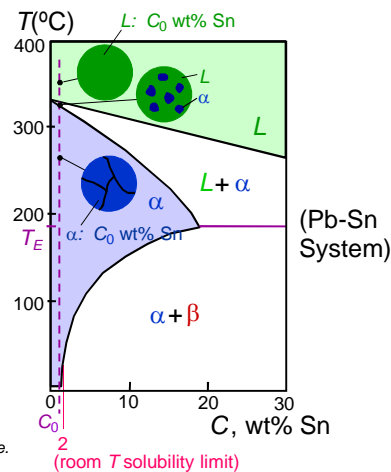


Adapted from Fig. 9.8, Callister & Rethwisch 8e.

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Microstructural Developments in Eutectic Systems I

- For alloys for which $C_0 < 2 \text{ wt\% Sn}$
- Result: at room temperature -- polycrystalline with grains of α phase having composition C_0

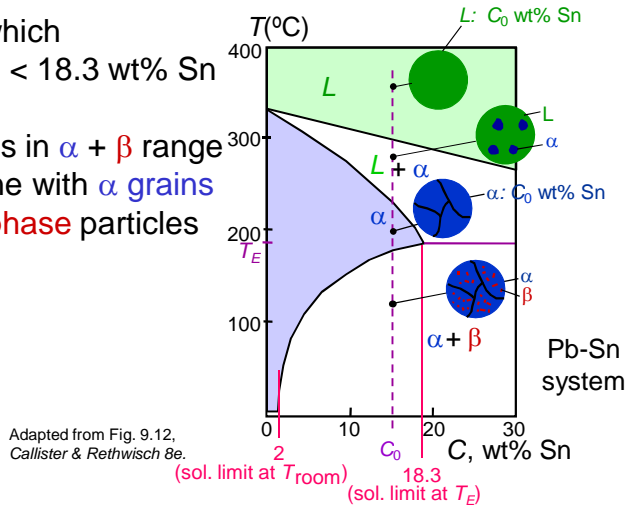


Adapted from Fig. 9.11, Callister & Rethwisch 8e.

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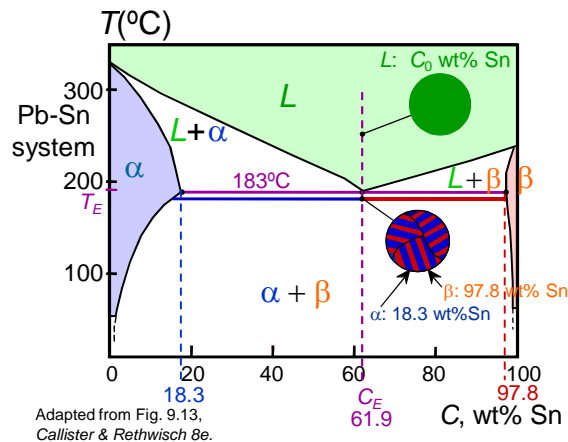
Microstructural Developments in Eutectic Systems II

- For alloys for which $2 \text{ wt\% Sn} < C_0 < 18.3 \text{ wt\% Sn}$
- Result: at temperatures in $\alpha + \beta$ range -- polycrystalline with α grains and small β -phase particles



Microstructural Developments in Eutectic Systems III

- For alloy of composition $C_0 = C_E$
- Result: Eutectic microstructure (lamellar structure) -- alternating layers (lamellae) of α and β phases.

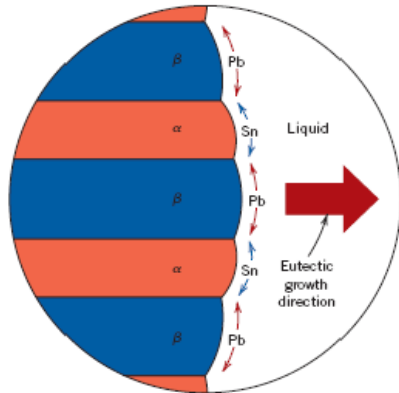


Micrograph of Pb-Sn eutectic microstructure

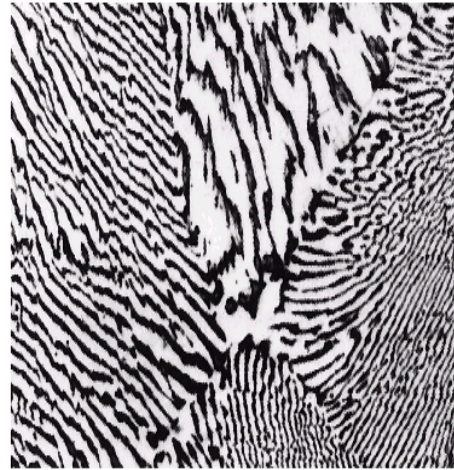


160 μm
Adapted from Fig. 9.14, Callister & Rethwisch 8e.

Lamellar Eutectic Structure



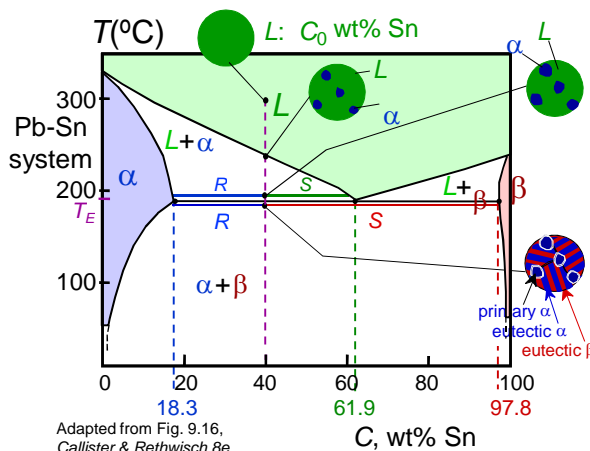
Redistribution of Pb and Sn occurs by short-distance diffusion of both elements.



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Microstructural Developments in Eutectic Systems IV

- For alloys for which $18.3 \text{ wt\% Sn} < C_0 < 61.9 \text{ wt\% Sn}$
- Result: α phase particles and an eutectic microconstituent



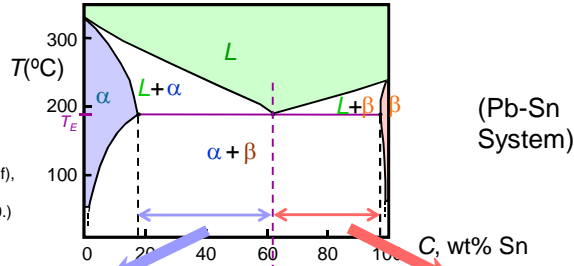
Adapted from Fig. 9.16, Callister & Rethwisch 8e.

- Just above T_E :
 - $C_\alpha = 18.3 \text{ wt\% Sn}$
 - $C_L = 61.9 \text{ wt\% Sn}$
 - $W_\alpha = \frac{S}{R+S} = 0.50$
 - $W_L = (1 - W_\alpha) = 0.50$
- Just below T_E :
 - $C_\alpha = 18.3 \text{ wt\% Sn}$
 - $C_\beta = 97.8 \text{ wt\% Sn}$
 - $W_\alpha = \frac{S}{R+S} = 0.73$
 - $W_\beta = 0.27$

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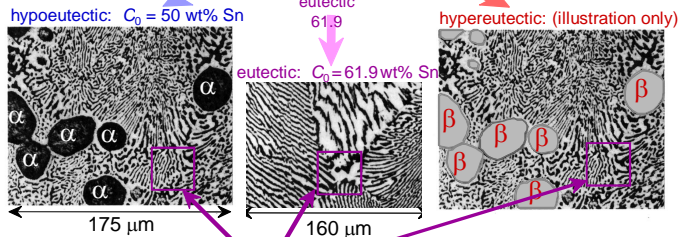
Hypoeutectic & Hypereutectic

Adapted from Fig. 9.8, Callister & Rethwisch 8e. (Fig. 10.8 adapted from *Binary Phase Diagrams*, 2nd ed., Vol. 3, T.B. Massalski (Editor-in-Chief), ASM International, Materials Park, OH, 1990.)



(Pb-Sn System)

(Figs. 9.14 and 9.17 from *Metals Handbook*, 9th ed., Vol. 9, *Metallography and Microstructures*, American Society for Metals, Materials Park, OH, 1985.)

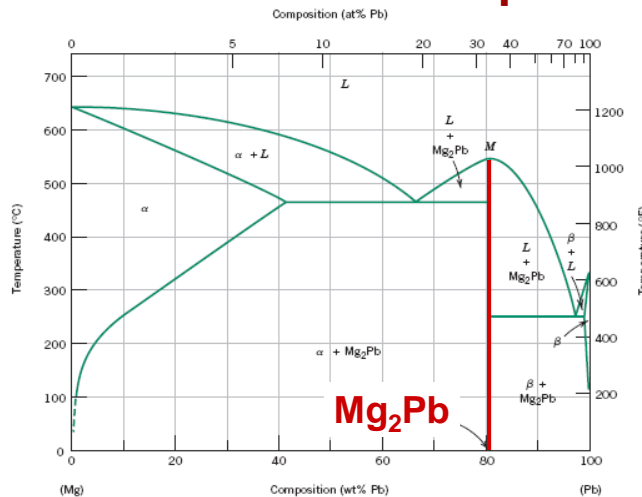


Adapted from Fig. 9.17, Callister & Rethwisch 8e.

Adapted from Fig. 9.14, Callister & Rethwisch 8e.

Adapted from Fig. 9.17, Callister & Rethwisch 8e. (Illustration only) Chapter 9 - 25

Intermetallic Compounds



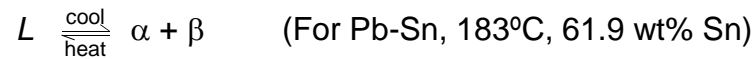
Adapted from Fig. 9.20, Callister & Rethwisch 8e.

Note: intermetallic compound exists as a line on the diagram - not an area - because of stoichiometry (i.e. composition of a compound is a fixed value).

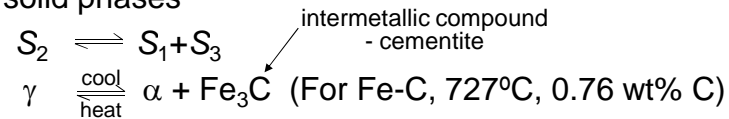
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Eutectic, Eutectoid, & Peritectic

- **Eutectic** - liquid transforms to two solid phases



- **Eutectoid** – one solid phase transforms to two other solid phases



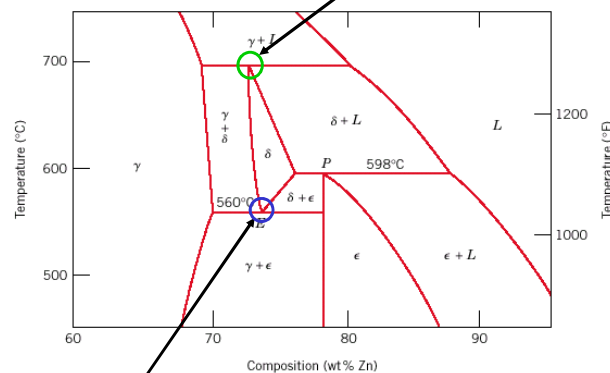
- **Peritectic** - liquid and one solid phase transform to a second solid phase



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Eutectoid & Peritectic

Cu-Zn Phase diagram

Peritectic transformation $\gamma + L \rightleftharpoons \delta$ Eutectoid transformation $\delta \rightleftharpoons \gamma + \epsilon$ Adapted from Fig. 9.21,
Callister & Rethwisch 8e.

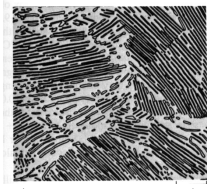
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Iron-Carbon (Fe-C) Phase Diagram

- 2 important points

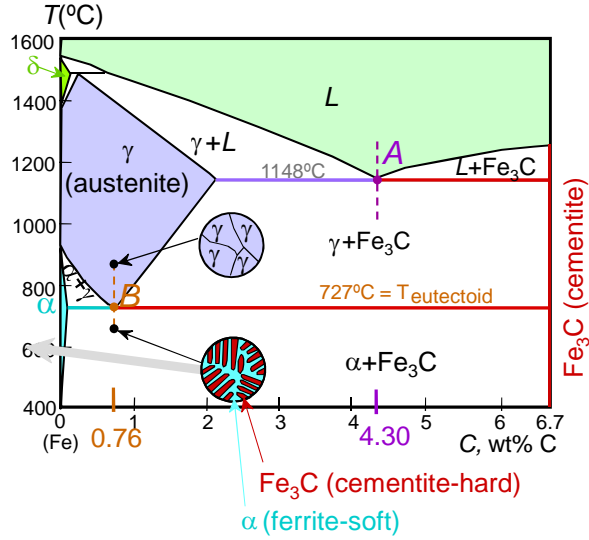
- Eutectic (A):
 $L \Rightarrow \gamma + Fe_3C$

- Eutectoid (B):
 $\gamma \Rightarrow \alpha + Fe_3C$



Result: Pearlite = alternating layers of α and Fe_3C phases

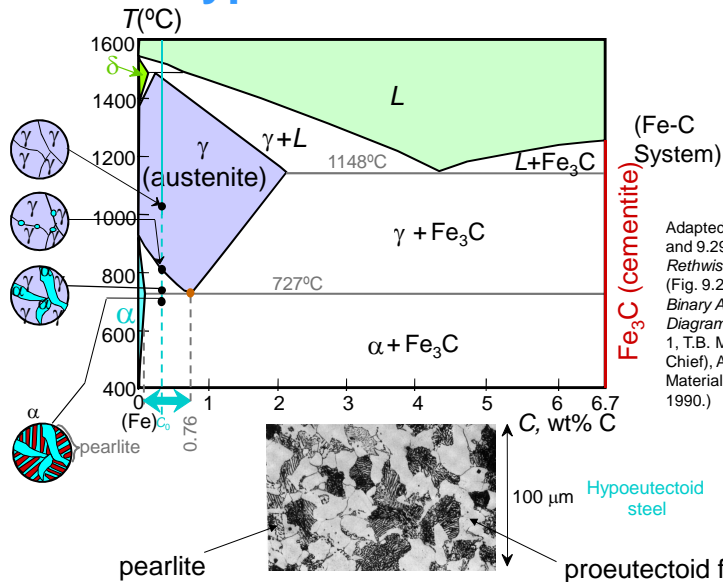
(Adapted from Fig. 9.27, Callister & Rethwisch 8e.)



Adapted from Fig. 9.24, Callister & Rethwisch 8e.

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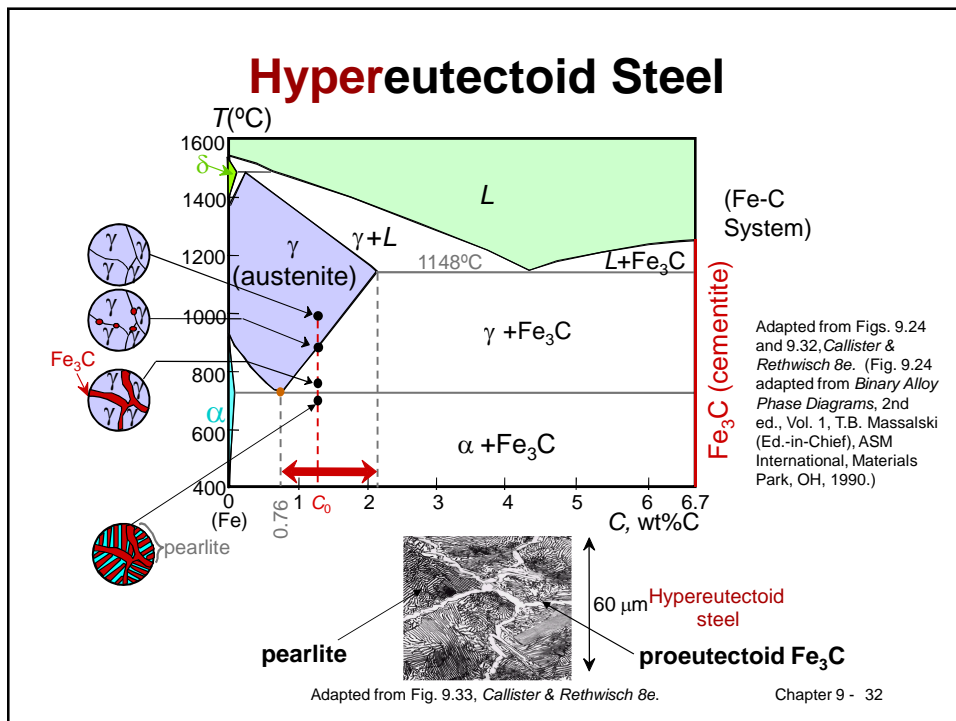
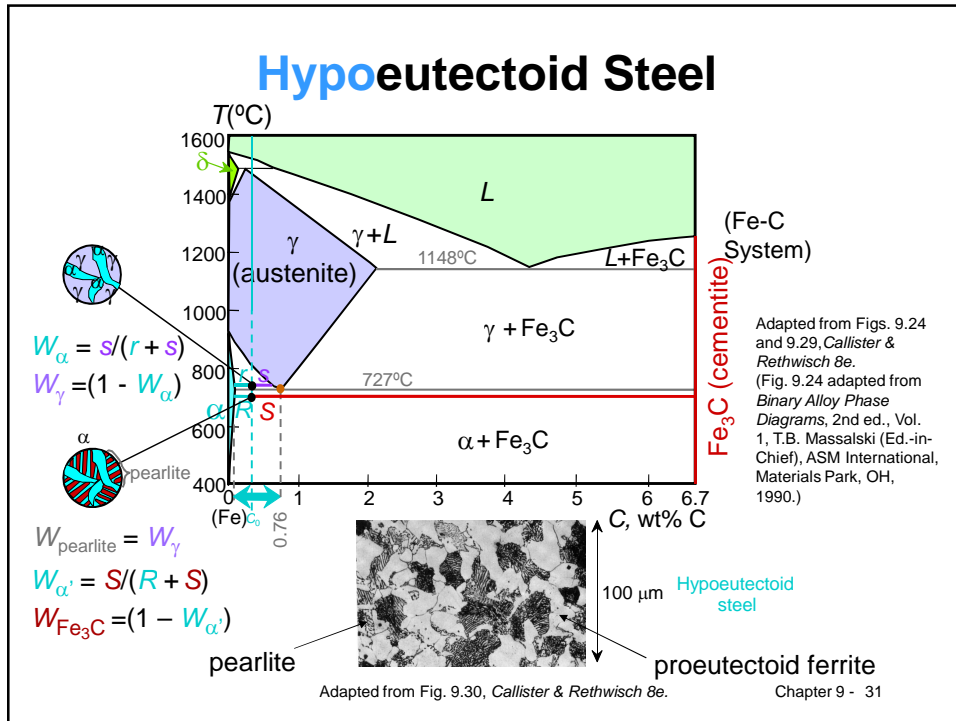
Hypoeutectoid Steel

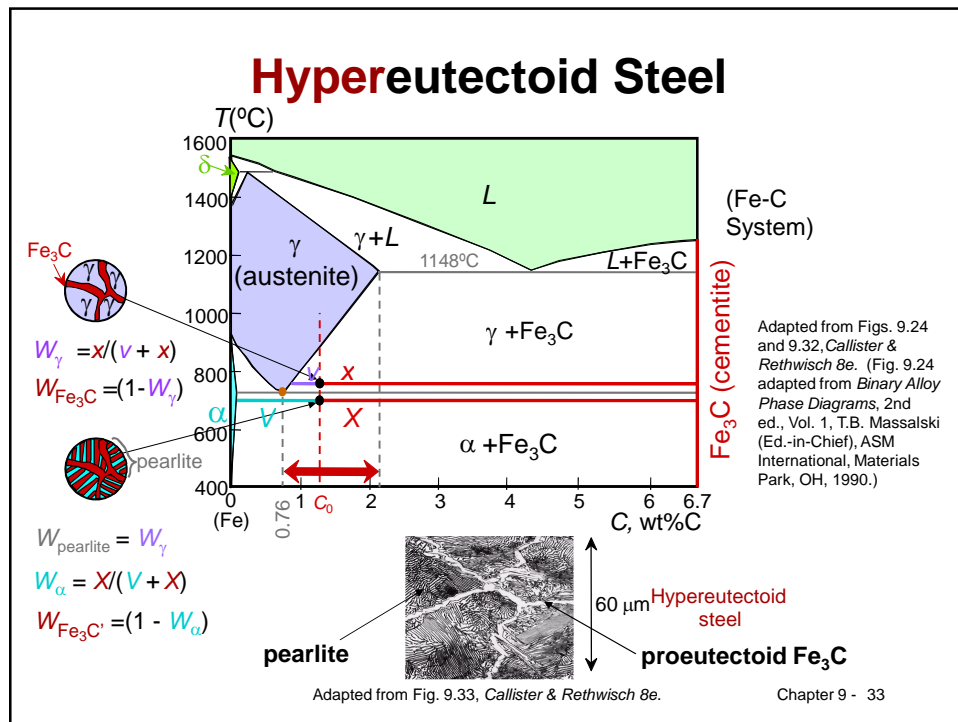


Adapted from Figs. 9.24 and 9.29, Callister & Rethwisch 8e. (Fig. 9.24 adapted from Binary Alloy Phase Diagrams, 2nd ed., Vol. 1, T.B. Massalski (Ed.-in-Chief), ASM International, Materials Park, OH, 1990.)

Adapted from Fig. 9.30, Callister & Rethwisch 8e.

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Example Problem

For a 99.6 wt% Fe-0.40 wt% C steel at a temperature just below the eutectoid, determine the following:

- The compositions of Fe_3C and ferrite (α).
- The amount of cementite (in grams) that forms in 100 g of steel.
- The amounts of pearlite and proeutectoid ferrite (α) in the 100 g.

Solution to Example Problem

a) Using the RS tie line just below the eutectoid

$$C_{\alpha} = 0.022 \text{ wt\% C}$$

$$C_{\text{Fe}_3\text{C}} = 6.70 \text{ wt\% C}$$

b) Using the lever rule with the tie line shown

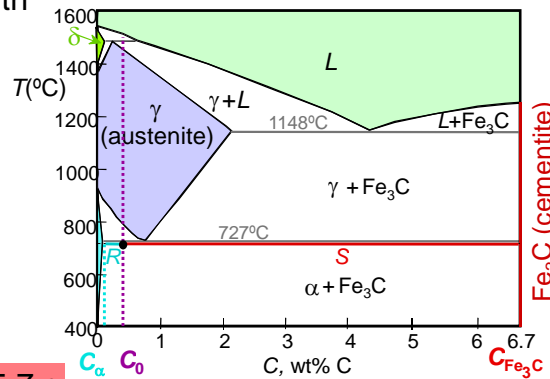
$$W_{\text{Fe}_3\text{C}} = \frac{R}{R+S} = \frac{C_0 - C_{\alpha}}{C_{\text{Fe}_3\text{C}} - C_{\alpha}}$$

$$= \frac{0.40 - 0.022}{6.70 - 0.022} = 0.057$$

Amount of Fe₃C in 100 g

$$= (100 \text{ g}) W_{\text{Fe}_3\text{C}}$$

$$= (100 \text{ g})(0.057) = \mathbf{5.7 \text{ g}}$$



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Solution to Example Problem (cont.)

c) Using the VX tie line just above the eutectoid and realizing that

$$C_0 = 0.40 \text{ wt\% C}$$

$$C_{\alpha} = 0.022 \text{ wt\% C}$$

$$C_{\text{pearlite}} = C_{\gamma} = 0.76 \text{ wt\% C}$$

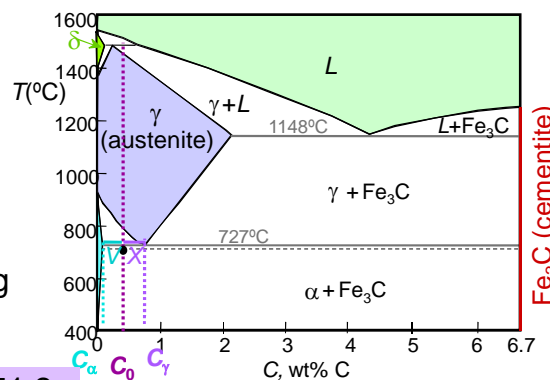
$$W_{\text{pearlite}} = \frac{V}{V+X} = \frac{C_0 - C_{\alpha}}{C_{\gamma} - C_{\alpha}}$$

$$= \frac{0.40 - 0.022}{0.76 - 0.022} = 0.512$$

Amount of pearlite in 100 g

$$= (100 \text{ g}) W_{\text{pearlite}}$$

$$= (100 \text{ g})(0.512) = \mathbf{51.2 \text{ g}}$$



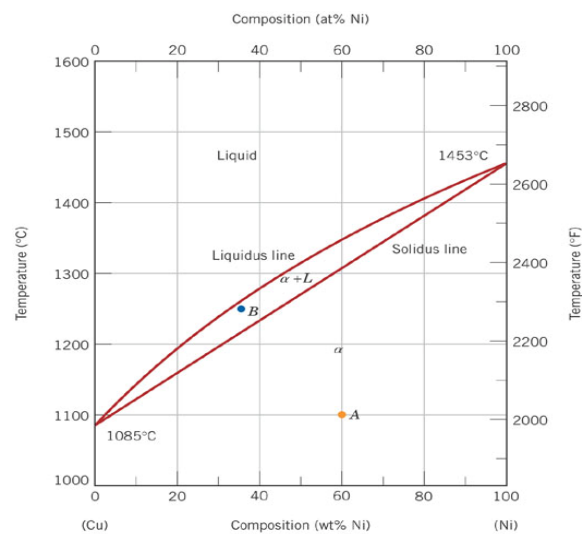
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Summary

- **Phase diagrams** are useful tools to determine:
 - the number and types of phases present,
 - the **composition** of each phase,
 - and the weight fraction of each phase
 given the temperature and composition of the system.
- The microstructure of an alloy depends on composition and cooling rate.
- Important phase diagram phase transformations include **eutectic**, **eutectoid**, and **peritectic**.

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Is it possible to have a copper–nickel alloy that, at equilibrium, consists of a liquid phase of composition 20 wt% Ni–80 wt% Cu and also an a phase of composition 39 wt% Ni–63 wt% Cu? If so, what will be the approximate temperature of the alloy? If this is not possible, explain why.



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For an iron-carbon alloy of composition 5 wt% C-95 wt% Fe, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 1175°C (2150°F), 1145°C (2095°F), and 700°C (1290°F). Label the phases and indicate their compositions (approximate).

