Örnek:

$$f(x,y) = \begin{cases} c\cdot e^{-2x}, & 0<y<x<\infty \\ 0, & d.h. \end{cases}$$

a.) $c = ?$, $P(X>Y) = ?$, $f_{XY} = ?$

$$\int_{x=0}^{\infty}\int_{y=0}^{x} c\cdot e^{-2x}\cdot dy\cdot dx = 1 \quad \text{olmalı.}$$

$$= \int_0^\infty c\cdot e^{-2x}\cdot y\Big|_0^x \cdot dx$$

$$= c\cdot \int_{x=0}^{\infty} \underbrace{x}_{u}\cdot \underbrace{e^{-2x}\cdot dx}_{dv}$$

$x = u$
$dx = du$
$e^{-2x}dx = dv$
$v = -\dfrac{e^{-2x}}{2}$

$$= c\cdot\Big[\underbrace{x\cdot\frac{e^{-2x}}{-2}\Big|_0^\infty}_{=0} + \int_0^\infty \frac{e^{-2x}}{2}\cdot dx$$

$$= c\cdot\left(-\frac{1}{4}\cdot e^{-2x}\right)\Big|_0^\infty = \frac{c}{4} = 1 \Rightarrow c = 4$$

$$\Longrightarrow f(x,y) = \begin{cases} 4\cdot e^{-2x}, & 0<y<x<\infty \\ 0, & d.h. \end{cases}$$

Ayrıca,

$$f_X(x) = \int_{y=0}^{x} 4\cdot e^{-2x}\cdot dy = 4\cdot e^{-2x}\cdot y\Big|_0^x = 4\cdot x\cdot e^{-2x}$$

$$f_Y(y) = \int_{x=y}^{\infty} 4\cdot e^{-2x}\cdot dx = -\frac{4}{2}\cdot e^{-2x}\Big|_{y}^{\infty} = 2\cdot e^{-2y}$$

$$P(X>Y) = P(X>Y \mid Y=y) = \int_{y\in D_Y} P(X>Y|Y=y)\, f_Y(y)\cdot dy$$

$$= \int_{y=0}^{\infty} \underline{P(X>y)}\cdot f_Y(y)\cdot dy = \int_{y=0}^{\infty} 2\cdot e^{-2y}\left[\int_{x=y}^{\infty} f_X(x)\,dx\right]$$

$$= \int_{y=0}^{\infty} 2\cdot e^{-2y}\cdot\left(\int_{x=y}^{\infty} 4x\cdot e^{-2x}\,dx\right)dy = \int_{y=0}^{\infty} 2\cdot e^{-2y}\left(2ye^{-2y} + e^{-2y}\right)$$

$$= \frac{3}{4}$$

b.) X, Y nin o.o.y.f. si

$$f(x,y) = \begin{cases} 2 \cdot e^{-2x-y} & , x > 0, y > 0 \\ 0 & , d.h. \end{cases}$$

şeklinde verildiğinde, Marjinaller.

$$f_X(x) = \int_{y=0}^{\infty} 2 \cdot e^{-2x-y} \, dy$$

$$= -2 \cdot e^{-2x-y} \Big|_{y=0}^{\infty} = 0 + 2 \cdot e^{-2x} = 2 \cdot e^{-2x},$$

$$f_Y(y) = \int_{x=0}^{\infty} 2 \cdot e^{-2x-y} \cdot dx = -e^{-2x-y} \Big|_{x=0}^{\infty}$$

$$= 0 + e^{-y}, \qquad f_Y(y) = e^{-y}, \quad y > 0$$

$\forall x, y \in \mathbb{R}$ için

$$f(x,y) = f_X(x) \cdot f_Y(y)$$

$$\Rightarrow 2 \cdot e^{-2x-y} = 2 \cdot e^{-2x} \cdot e^{-y}$$

olup X, Y bağımsız t.d. lerdir.

Böylece, $p(X > Y) = \int_{y=0}^{\infty} p(X > y) \cdot f_Y(y) \cdot dy$

Burada, $p(X > y) = \int_{x=y}^{\infty} f_X(x) \cdot dx$

$$= \int_{x=y}^{\infty} 2 \cdot e^{-2x} \, dx = e^{-2y}$$

$$= \int_{y=0}^{\infty} e^{-2y} \cdot e^{-y} \cdot dy = \int e^{-3y} \, dy = -\frac{e^{-3y}}{3} \Big|_{0}^{\infty} = \frac{1}{3}$$

olur.