

Örnek: x_1, x_2, x_3 bağımsız $U(0,1)$ dağılımından $n=3$ birimlik örneklem.

$y_1 = x(1), y_2 = x(2), y_3 = x(3)$ bu örneklemde karşılıklı gelen sırasıyla olsun. $Z = y_3 - y_1$ t.d. nin $f(z), v(z) = ?$

$$f(x) = \begin{cases} 1 & ; 0 < x < 1 \\ 0 & ; \text{diğer} \end{cases}$$

$$F(x) = P(X \leq x) = \int_0^x 1 \cdot dx = x \Big|_0^x = x$$

$$f_{y(1), y(3)} = \frac{n!}{0! \cdot 1! \cdot 0!} \cdot f(x) \cdot f(y) \cdot [F(x)]^0 \cdot [F(y) - F(x)]^{3-1-1} \cdot [1 - F(y)]^0$$

$$= 3! \cdot 1 \cdot 1 \cdot [y - x]^1 = 6 \cdot (y - x), 0 < x < y < 1$$

$(Z = y_3 - y_1 = x(3) - x(1))$ örneklem genişliği için olur.
 $(V = \frac{x(3) + x(1)}{2})$ tanımlayalım.

$$\Rightarrow \left. \begin{aligned} x(1) &= y_1 = V - \frac{z}{2} \\ x(3) &= y_3 = V + \frac{z}{2} \end{aligned} \right\} \text{Pers dönüşüm olur.}$$

$$J = \begin{bmatrix} \frac{\partial x(1)}{\partial v} & \frac{\partial x(1)}{\partial z} \\ \frac{\partial x(3)}{\partial v} & \frac{\partial x(3)}{\partial z} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ 1 & \frac{1}{2} \end{bmatrix} \quad \checkmark$$

$$\Rightarrow |J| = \frac{1}{2} + \frac{1}{2} = 1$$

$$\Rightarrow f_{z,v}(x(z,v), y(z,v)) = 1 \cdot 6 \cdot \left[\left(V + \frac{z}{2} \right) - \left(V - \frac{z}{2} \right) \right]$$

$$= 6z, \quad 0 < z < 1$$

buradan z 'nin marginal yoğ. fonk.

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$$f_{\mathbb{Z}}(z) = \int f_{z,v}(z,v) \cdot dv$$

$$= \int_{v=\frac{z}{2}}^{\frac{1-z}{2}} 6z \cdot dv = 6z \cdot v \Big|_{\frac{z}{2}}^{\frac{1-z}{2}}$$

$$= 6z \cdot \left(1 - \frac{z}{2} - \frac{z}{2}\right) = 6z \cdot (1-z), \quad 0 < z < 1$$

elde edilir.

$$\Rightarrow \mathbb{E}(z) = \int_0^1 z \cdot 6z \cdot (1-z) dz$$

$$= \int_0^1 6z^2 \cdot dz - \int_0^1 6z^3 \cdot dz$$

$$= \frac{6}{3} z^3 \Big|_0^1 - \frac{6}{4} z^4 \Big|_0^1 = \frac{6}{3} - \frac{6}{4} = \frac{1}{2} //$$

$$\mathbb{E}(z^2) = \int_0^1 z^2 \cdot f_z(z) dz$$

$$= \int_0^1 6z^3 \cdot dz - \int_0^1 6z^4 \cdot dz$$

$$= \frac{6}{4} z^4 \Big|_0^1 - \frac{6}{5} z^5 \Big|_0^1 = \frac{6}{4} - \frac{6}{5} = \frac{6}{20} //$$

$$\Rightarrow v(z) = \frac{6}{20} - \left(\frac{1}{2}\right)^2 = \frac{6}{20} - \frac{1}{4} = \frac{1}{20} //$$

(11)

elde edilir.