

Karekök Sayılar

Karekök içindeki sayı, bir sayının karesi ise karesi olan sayı olarak ayrılır.

Karekök içindeki sayı bir kare olup olmadığına bakılarak ayrılır.

$$\sqrt{20} = 2\sqrt{5}$$

$$\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$$

Not: Bazı sayıları ayırmakta güçlük çekerseniz OBEB, OKEB de ki gibi sayılara ayırabilirsiniz.

$$\begin{array}{r|l} 96 & 2 \\ 48 & 2 \\ 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} 2 \quad \sqrt{96} = 2^2 \sqrt{23} = 4\sqrt{6}$$

Örn: $\sqrt{6300} = ?$

$$\sqrt{63 \cdot 100} = 10\sqrt{63} = 10\sqrt{9 \cdot 7} = 10 \cdot 3\sqrt{7} = 30\sqrt{7}$$

Not: Reel sayılarla kareköküne iki negatif alamaz. Aynı zamanda tüm çift dereceli kökler içinde aynı durum geçerlidir.

$$\sqrt{-2} \notin \mathbb{R}$$

$$\sqrt[4]{-7} \notin \mathbb{R}$$

$$\sqrt{-6} \notin \mathbb{R}$$

$$\sqrt[8]{-4} \notin \mathbb{R}$$

Not: Tek dereceli tüm köklü sayılar reeldir.

$$\sqrt[3]{-3} = -\sqrt[3]{3} \in \mathbb{R}$$

$$\sqrt[5]{-7} = -\sqrt[5]{7} \in \mathbb{R}$$

Not: $\sqrt[n]{a}$ ifadesinde

n çift ise sonuç $|a|$

n tek ise sonuç a

Örn: $\sqrt[4]{(-2)^4} = |-2| = 2$

$$\sqrt[4]{-2^4} \notin \mathbb{R}$$

Not: Derisi ve üssü aynı olan köklü sayıların arasında toplama ve çıkarma yapılır.

$$a\sqrt{x} + b\sqrt{y} - c\sqrt{x} + d\sqrt{y} = (a-c)\sqrt{x} + (b+d)\sqrt{y}$$

Örn: $\sqrt{48} + 2\sqrt{75} - \sqrt{243} = ?$

$$\sqrt{16 \cdot 3} + 2\sqrt{25 \cdot 3} - \sqrt{81 \cdot 3}$$

$$4\sqrt{3} + 10\sqrt{3} - 9\sqrt{3} = 5\sqrt{3}$$

Örn: $\sqrt{(-4)^2} + \sqrt{4^2} - (-2)^3 = ?$

$$|-4| + 4 - (-8) = 8$$

Örn: $3\sqrt{8} + 2\sqrt{2} - (\sqrt{8} + \sqrt{2}) = ?$

$$3\sqrt{4 \cdot 2} + 2\sqrt{2} - (\sqrt{4 \cdot 2} + \sqrt{2})$$

$$6\sqrt{2} + 2\sqrt{2} - (2\sqrt{2} + \sqrt{2})$$

$$8\sqrt{2} - 3\sqrt{2} = 5\sqrt{2}$$

Not: $\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{x \cdot y}$

$\sqrt[n]{\frac{x}{y}} = \sqrt[n]{\frac{x}{y}}$

$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

$\sqrt{a} \cdot \sqrt{a} = a$

$x\sqrt{a} \cdot y\sqrt{b} = x \cdot y \sqrt{ab}$

$(a\sqrt{b})^2 = (a\sqrt{b}) \cdot (a\sqrt{b}) = a^2 \cdot b$

am: $\sqrt{3} \cdot \sqrt{6} = \sqrt{18} \Rightarrow \sqrt{9 \cdot 2} = 3\sqrt{2}$

$\sqrt{7} \cdot \sqrt{7} = 7$

$2\sqrt{2} \cdot 3\sqrt{6} = 6\sqrt{12} \Rightarrow 6\sqrt{4 \cdot 3} = 12\sqrt{3}$

$(2\sqrt{5})^2 = (2\sqrt{5}) \cdot (2\sqrt{5}) = 4 \cdot 5 = 20$

$(3\sqrt{2})^2 = 3^2 \cdot 2 = 18$

am: $(\sqrt{2} - \sqrt{5})^2 + 2\sqrt{10} + 3 = ?$

$= (\sqrt{2})^2 - 2\sqrt{10} + (\sqrt{5})^2 + 2\sqrt{10} + 3$

$= 2 + 5 + 3$

$= 10$

am: $\frac{\sqrt{4,44} + \sqrt{9,99}}{\sqrt{111}} = ?$

$\frac{\sqrt{\frac{444}{100}} + \sqrt{\frac{999}{100}}}{\sqrt{111}} = \frac{\sqrt{444} + \sqrt{999}}{10\sqrt{111}} = \frac{\sqrt{4 \cdot 111} + \sqrt{9 \cdot 111}}{10\sqrt{111}} = \frac{2\sqrt{111} + 3\sqrt{111}}{10\sqrt{111}}$

$= \frac{2\sqrt{111} + 3\sqrt{111}}{10\sqrt{111}} = \frac{\sqrt{111} \cdot (2+3)}{\sqrt{111} \cdot 10} = \frac{5}{10} = \frac{1}{2}$

$= 0,5$

Örn: $\sqrt{10} \cdot (\sqrt{64} + \sqrt{0,4}) = ?$

$$\sqrt{10} \cdot \left(\sqrt{\frac{64}{10}} + \sqrt{\frac{4}{10}} \right) \Rightarrow \sqrt{10} \cdot \frac{\sqrt{64} + \sqrt{4}}{\sqrt{10}} = 8 + 2 = 10$$

Örn: $a = \sqrt{2} + 1$ ise $a(a-1)(a-2) = ?$

$$\sqrt{2} + 1 \cdot \sqrt{2} \cdot \sqrt{2} - 1 \Rightarrow$$

$$(2-1)\sqrt{2} = \sqrt{2}$$

Örn: $\frac{\sqrt{40} \cdot \sqrt{18}}{\sqrt{80}} = ? \Rightarrow \sqrt{\frac{40 \cdot 18}{80}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$

Örn: $\frac{\sqrt{0,48} - \sqrt{0,27}}{\sqrt{1,47}} = ?$

$$\frac{\sqrt{\frac{48}{100}} - \sqrt{\frac{27}{100}}}{\sqrt{\frac{147}{100}}} = \frac{\frac{4\sqrt{3}}{10} - \frac{3\sqrt{3}}{10}}{\frac{\sqrt{49 \cdot 3}}{10}} = \frac{\frac{\sqrt{3}}{10}}{\frac{7\sqrt{3}}{10}} = \frac{1}{7}$$

Örn: $\sqrt{0,25} - \sqrt{1,21} + \sqrt{1,44} = ?$

$$\sqrt{\frac{25}{100}} - \sqrt{\frac{121}{100}} + \sqrt{\frac{144}{100}} = \frac{5}{10} - \frac{11}{10} + \frac{12}{10} = \frac{6}{10} = \frac{3}{5}$$

Örn: $\frac{\sqrt{3} \cdot \sqrt{12}}{\sqrt{0,16} + \sqrt{0,36}} = \frac{\sqrt{36}}{0,4 + 0,6} = \frac{6}{1} = 6$

Not: Paydadaki köktü ifade ile kuvv. gösterilir.

$$- \frac{4}{\frac{\sqrt{2}}{(\sqrt{2})}} \Rightarrow \frac{4 \cdot \sqrt{2}}{2} = 2\sqrt{2}$$

$$- \frac{1}{\frac{2\sqrt{3}}{(\sqrt{3})}} = \frac{\sqrt{3}}{2 \cdot 3} = \frac{\sqrt{3}}{6}$$

$$- \frac{1}{\frac{1-\sqrt{2}}{(1+\sqrt{2})}} = \frac{1\sqrt{2}}{1-2} = -1-\sqrt{2}$$

$$- \frac{3-\sqrt{2}}{\frac{2-\sqrt{2}}{(2+\sqrt{2})}} = \frac{6-2\sqrt{2}+3\sqrt{2}-2}{4-2} = \frac{4+\sqrt{2}}{2} = 2 + \frac{\sqrt{2}}{2}$$

örn: a) $\sqrt{2}+1$

Hangisi rasyonel sayıdır?

b) $2\sqrt{2}-1$

c) $\frac{1}{\frac{\sqrt{2}}{(\sqrt{2})}} = \frac{\sqrt{2}}{2}$

d) $\frac{\sqrt{2}}{\frac{\sqrt{2}+1}{(\sqrt{2}-1)}} = \frac{2-\sqrt{2}}{2-1} = \frac{2-\sqrt{2}}{1}$

e) $\frac{2\sqrt{2}-2}{3\sqrt{2}-3} = \frac{12-6\sqrt{2}+6\sqrt{2}-6}{18-9} = \frac{6}{9} = \left(\frac{2}{3}\right)\sqrt{\quad}$

örn: $\frac{1}{\frac{\sqrt{2}+1}{(\sqrt{2}-1)}} - \frac{1}{\frac{\sqrt{2}-1}{(\sqrt{2}+1)}} = ?$

$$\frac{\sqrt{2}-1}{2-1} - \frac{\sqrt{2}+1}{2-1} \Rightarrow \frac{\sqrt{2}-1}{1} - \left(\frac{\sqrt{2}+1}{1}\right) \Rightarrow \sqrt{2}-1-\sqrt{2}-1 = -2$$

Örn: $\frac{\sqrt{2} - \frac{1}{\sqrt{2}}}{\sqrt{2} + \frac{1}{\sqrt{2}}} = ?$ $\frac{\frac{1}{\sqrt{2}}}{\frac{3}{\sqrt{2}}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{3} = \frac{1}{3}$

Örn: $a = \sqrt{b} + 1$

$b = \sqrt{b} - 1$

$\frac{a}{b} + \frac{b}{a} = ?$

$$\frac{\sqrt{b}+1}{\sqrt{b}-1} + \frac{\sqrt{b}-1}{\sqrt{b}+1} \Rightarrow \frac{b+2\sqrt{b}+1}{5} + \frac{b-2\sqrt{b}+1}{5} = \frac{b+2\sqrt{b}+1+b-2\sqrt{b}+1}{5} = \frac{14}{5}$$

Örn: $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \sqrt{ab}$ ise b 'nin a türünden değeri nedir?

$$\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{ab} \Rightarrow \frac{a}{\sqrt{ab}} + \frac{b}{\sqrt{ab}} = \sqrt{ab} \Rightarrow \frac{a+b}{\sqrt{ab}} \cdot \frac{\sqrt{ab}}{1} \Rightarrow a+b = ab$$

$$a = ab - b$$

$$a = b(a-1)$$

$$b = \frac{a}{a-1}$$

Köklere Devir Kuralları

$$\sqrt[n]{x} = \sqrt[n]{x^m}$$

$$m \cdot p \sqrt[n]{x^{m \cdot n}} = \sqrt[n]{x^m}$$

Örn: $\sqrt[3]{2} \cdot \sqrt{2} = ?$

$$\sqrt[3]{2} \cdot \sqrt{2} \Rightarrow \sqrt[3]{2^2} \cdot \sqrt[3]{2^3} = \sqrt[6]{2^2} \cdot \sqrt[6]{2^3} = \sqrt[6]{2^5} = \sqrt[6]{32}$$

Not: $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Örn: $\sqrt[3]{2^6} = 2^{\frac{6}{3}} = 2^2 = 4$

$$\sqrt{3} = 3^{\frac{1}{2}}$$

$$\sqrt[5]{4} = 4^{\frac{1}{5}}$$

Qm: $3^{-\frac{1}{2}} \cdot \sqrt{27} = ?$

$$\frac{1}{3^{\frac{1}{2}}} \cdot \sqrt{27} = \frac{1}{\sqrt{3}} \cdot \sqrt{27} = \frac{\sqrt{27}}{\sqrt{3}} = \sqrt{9} = 3$$

Qm: $\sqrt[4]{0.0256} \cdot \sqrt[3]{(0.008)^{-1}} = ?$

$$\sqrt[4]{\frac{256}{10000}} \cdot \sqrt[3]{\frac{1000}{8}} \Rightarrow \sqrt[4]{\frac{4^4}{10^4}} \cdot \sqrt[3]{\frac{10^3}{2^3}} = \frac{4}{10} \cdot \frac{10}{2} = 2$$

Qm: $\frac{5\sqrt{3^{5x-10}}}{\sqrt{9^{2x-1}}} = \frac{1}{27}$

$x = ?$

$$\frac{3^{\frac{5x-10}{2}}}{9^{\frac{2x-1}{2}}} = \frac{1}{27} \Rightarrow \frac{3^{x-2}}{9^{\frac{2x-1}{2}}} = 3^{-3} \Rightarrow \frac{3^{x-2}}{3^{2x-1}} \times 3^{-3} \Rightarrow 3^{x-2} = 3^{2x-1} \cdot 3^{-3}$$

$$3^{x-2} = 3^{2x-4}$$

$$x-2 = 2x-4$$

$$x = 2$$

Qm: $\frac{\sqrt{9^{3x+1}}}{\sqrt[3]{3^{9x-34}}} = 27$

in $y = ?$

$$\frac{9^{\frac{3x+1}{2}}}{3^{\frac{9x-34}{3}}} = 3^3 \Rightarrow \frac{3^{3x+1}}{3^{3x-11}} \times 3^3 \Rightarrow 3^{x+1} = 3^{3x-8} \cdot 3^3$$

$$\Rightarrow 3^{x+1} = 3^{3x-5}$$

$$1 = -y + 3$$

$$y = 2$$

Not: $a^n \sqrt[n]{x} = \sqrt[n]{a^n \cdot x}$

örn: $2^3 \sqrt{3} = \sqrt[3]{2^3 \cdot 3} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{24}$

örn: $2^3 \sqrt{12} = \sqrt[3]{36 \cdot a}$ ise $a = ?$

$$3 \sqrt{8 \cdot 12} = \sqrt[3]{36 \cdot a} \quad a = \frac{8 \cdot 12}{36} = \frac{8}{3}$$

örn: $x \cdot \sqrt{0,4} = 1$ ise $x = ?$

$$x \cdot \sqrt{\frac{4}{10}} = 1 \Rightarrow \sqrt{x^2 \cdot \frac{4}{10}} = \sqrt{1} \Rightarrow x^2 \cdot \frac{4}{10} = 1 \Rightarrow x^2 = \frac{10}{4} \Rightarrow x = \sqrt{\frac{5}{2}}$$

Not: if ise köklr

* $\sqrt[n]{\sqrt[n]{x}} = \sqrt[n \cdot n]{x}$

* $\sqrt[n]{\sqrt[m]{\sqrt[k]{x}}} = \sqrt[m \cdot n \cdot k]{x}$

* $\sqrt[n]{x^m \sqrt[k]{y}} = \sqrt[n \cdot m]{x^m} \sqrt[n \cdot k]{y}$

* $\sqrt[m]{x^2 \sqrt[k]{y^4 \sqrt[n]{z}}} = \sqrt[m \cdot n \cdot k]{x^{2n} y^{4m} z^m}$

örn: $\sqrt{2^3 \sqrt{2}} = \sqrt[6]{2^3 \cdot 2} = \sqrt[6]{16}$

örn: $3 \sqrt{2^3 \sqrt{2} \sqrt{2}} = \sqrt[18]{2^6 \cdot 2^2 \cdot 2} = \sqrt[18]{2^3}$

örn: $3 \sqrt{2^5 \sqrt{x}} = \sqrt[3]{2^5} \cdot \sqrt[5]{x} \quad x = ?$

$$\sqrt[15]{2^5 \cdot x} = \sqrt[15]{2^5} \cdot \sqrt[15]{x}$$

$$\sqrt[15]{2^5 \cdot x} = \sqrt[15]{2^5 \cdot 3^3}$$

$$2^5 \cdot x = 2^5 \cdot 3^3$$

$$x = 3^3 \Rightarrow x = 27$$

Not: $x, y \in \mathbb{R}^+$, $x > y$, $a = x + y$, $b = x \cdot y$ ise

$$\sqrt{a \pm 2\sqrt{b}} = \sqrt{x} \pm \sqrt{y}$$

örn: $\sqrt{10 + 2\sqrt{21}} = ?$

$\begin{matrix} \swarrow & \searrow \\ 3 & 7 \end{matrix}$

$$= \sqrt{3} + \sqrt{7}$$

örn: $\sqrt{5 - \sqrt{24}} = ?$

$\begin{matrix} \swarrow & \searrow \\ 3 & 2 \end{matrix}$

$$\sqrt{5 - \sqrt{6 \cdot 4}} \Rightarrow \sqrt{5 - 2\sqrt{6}} \Rightarrow \sqrt{3} - \sqrt{2}$$

$\begin{matrix} \swarrow & \searrow \\ 3 & 2 \end{matrix}$

örn: $\sqrt{4 + \sqrt{15}} = ?$

$$\sqrt{4 + \sqrt{4 \cdot \frac{15}{4}}} = \sqrt{4 + 2\sqrt{\frac{15}{4}}} \Rightarrow \sqrt{\frac{5}{2}} + \sqrt{\frac{3}{2}}$$

$\begin{matrix} \swarrow & \searrow \\ \frac{5}{2} & \frac{3}{2} \end{matrix}$

Not: Aritmetik ortalaması = $\frac{\text{Terimlerin toplamı}}{\text{Terim sayısı}}$

$$\sqrt{6 + 2\sqrt{5}}, \sqrt{6 - 2\sqrt{5}} \text{ in A.O} = ?$$

$\begin{matrix} \swarrow & \searrow \\ 5 & 1 \end{matrix}$ $\begin{matrix} \swarrow & \searrow \\ 5 & 1 \end{matrix}$

$$A.O = \frac{\sqrt{5+1} + \sqrt{5-1}}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$$

örn: $\sqrt{4 + 2\sqrt{3}} - \sqrt{4 - 2\sqrt{3}} = ?$

$\begin{matrix} \swarrow & \searrow \\ 3 & 1 \end{matrix}$ $\begin{matrix} \swarrow & \searrow \\ 3 & 1 \end{matrix}$

$$(\sqrt{3+1}) - (\sqrt{3-1}) = \cancel{\sqrt{3}} + \sqrt{1} - \cancel{\sqrt{3}} + \sqrt{1} = 2\sqrt{1} = 2$$

Not: $\sqrt[n]{f(x)}$ ifadesinde;

- n tek ise ifade her zaman reel sayıdır.
- n çift ise $f(x) \geq 0$ iis reel sayıdır.

Örn: $\sqrt{3x-9} + \sqrt[4]{7-x}$ reel sayı ise x hangi aralıktadır?

$$3x-9 \geq 0 \rightarrow 3x \geq 9 \rightarrow x \geq 3$$

$$7-x \geq 0 \rightarrow 7 \geq x$$

$$7 \geq x \geq 3$$

$$x = \{7, 6, 5, 4, 3\}$$

Örn: $\sqrt{\frac{9}{25} - \frac{6}{35} + \frac{1}{49}} = ?$

$$\left(\frac{3}{5}\right)^2 - 2\left(\frac{3}{5}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2 \Rightarrow \sqrt{\left(\frac{3}{5} - \frac{1}{7}\right)^2} = \left|\frac{3}{5} - \frac{1}{7}\right| = \frac{21}{35} - \frac{5}{35} = \frac{16}{35}$$

Örn: $\frac{\sqrt{15} + \sqrt{5} - \sqrt{6} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} = ? \Rightarrow \frac{\sqrt{3} \cdot \sqrt{5} + \sqrt{5} - \sqrt{3} \cdot \sqrt{2} - \sqrt{2}}{\sqrt{5} - \sqrt{2}}$

$$= \frac{\sqrt{5}(\sqrt{3}+1) - \sqrt{2}(\sqrt{3}+1)}{\sqrt{5} - \sqrt{2}} = \frac{(\sqrt{3}+1)(\sqrt{5}-\sqrt{2})}{\sqrt{5}-\sqrt{2}} = \sqrt{3}+1$$

Örn: $\sqrt{x+5} - \sqrt{x} = m$ ise $\sqrt{x+5} + \sqrt{x}$ nin m üssünde değeri?

$$\sqrt{x+5} - \sqrt{x} = m$$

$$\sqrt{x+5} + \sqrt{x} = n$$

$$x+5-x = m \cdot n \Rightarrow 5 = m \cdot n \quad n = \frac{5}{m}$$

Örn: $\sqrt{7} - \sqrt{2} = A$ ise $\sqrt{7} + \sqrt{2}$ nin A üssünde değeri?

$$A \cdot n = (\sqrt{7} - \sqrt{2}) \cdot (\sqrt{7} + \sqrt{2})$$

$$A \cdot n = 7 - 2$$

$$A \cdot n = 5 \quad n = \frac{5}{A}$$

Net: $\sqrt{30 + \sqrt{30 + \sqrt{30} + \dots}}$ tentukan apakah keoma x dan lipit x'2
isithorir.

$$\sqrt{30+x} = x$$

$$30 + x = x^2$$

$$\begin{array}{r} x^2 - x - 30 = 0 \\ x \quad -6 \\ x \quad +5 \end{array}$$

$$L.S = \{x=6, x=-5\}$$

örn: $\frac{\sqrt{72 - \sqrt{72 - \sqrt{72} - \dots}}}{9.6} + \frac{\sqrt{42 + \sqrt{42 + \sqrt{42} + \dots}}}{6.7} = ?$

$$\sqrt{72-x} = x$$

$$72-x = x^2$$

$$\begin{array}{r} x^2 + x - 72 = 0 \\ x \quad +9 \\ x \quad -8 \end{array}$$

$$\begin{array}{l} x = -9 \\ x = +8 \end{array}$$

$$\sqrt{42+x} = x$$

$$42+x = x^2$$

$$\begin{array}{r} x^2 - x - 42 = 0 \\ x \quad -7 \\ x \quad +6 \end{array}$$

$$\begin{array}{l} x = +7 \\ x = -6 \end{array}$$

Net: $\sqrt[n]{a \sqrt[n]{a \sqrt[n]{a} \dots}} = \sqrt[n-1]{a}$

$$\sqrt[n]{a \sqrt[n]{a \sqrt[n]{a} \dots}} = \sqrt[n-1]{a}$$

örn: $\sqrt[8]{2 \sqrt[2]{2 \sqrt[2]{2} \dots}} = \sqrt{2}$

örn: $2 \sqrt{a \sqrt{a \sqrt{a} \dots}} - \sqrt[3]{a \sqrt[3]{a \sqrt[3]{a} \dots}} = 6$ dan $0 \in R$ dan $0 = ?$

$$2 \sqrt[2]{a} - \sqrt[3]{a} = 6$$

$$2a - \sqrt{a} = 6$$

$$(2a-6)^2 = (\sqrt{a})^2$$

$$4a^2 - 24a + 36 = a$$

$$4a^2 - 25a + 36 = 0$$

$$\begin{array}{r} 4 \\ 1 \end{array} \quad \begin{array}{r} -9 \\ -4 \end{array}$$

$$4a - 9 = 0 \rightarrow \frac{9}{4}$$

$$0 - 4 = 0 \rightarrow 4 \checkmark$$

$$2a - 6 = \sqrt{a}$$

$$\frac{9}{2} - 6 = \frac{9}{3}$$

$$\frac{9}{2} - 6 = \frac{3}{2}$$

$$\frac{6}{2} - 6 = 0$$