

but, because  $\epsilon = \sigma/E$ ,

$$\frac{\sigma}{E_{ct}} = \frac{\sigma}{E_m} V_m + \frac{\sigma}{E_f} V_f \quad (16.14)$$

where  $E_{ct}$  is the modulus of elasticity in the transverse direction. Now, dividing through by  $\sigma$  yields

$$\frac{1}{E_{ct}} = \frac{V_m}{E_m} + \frac{V_f}{E_f} \quad (16.15)$$

which reduces to

$$E_{ct} = \frac{E_m E_f}{V_m E_f + V_f E_m} = \frac{E_m E_f}{(1 - V_f) E_f + V_f E_m} \quad (16.16)$$

For a continuous and aligned fiber-reinforced composite, modulus of elasticity in the transverse direction

Equation 16.16 is analogous to the lower-bound expression for particulate composites, Equation 16.2.

### EXAMPLE PROBLEM 16.2

#### Elastic Modulus Determination for a Glass Fiber-Reinforced Composite—Transverse Direction

Compute the elastic modulus of the composite material described in Example Problem 16.1, but assume that the stress is applied perpendicular to the direction of fiber alignment.

#### Solution

According to Equation 16.16,

$$\begin{aligned} E_{ct} &= \frac{(3.4 \text{ GPa})(69 \text{ GPa})}{(0.6)(69 \text{ GPa}) + (0.4)(3.4 \text{ GPa})} \\ &= 5.5 \text{ GPa } (0.81 \times 10^6 \text{ psi}) \end{aligned}$$

This value for  $E_{ct}$  is slightly greater than that of the matrix phase but, from Example Problem 16.1a, only approximately one-fifth of the modulus of elasticity along the fiber direction ( $E_{cl}$ ), which indicates the degree of anisotropy of continuous and oriented fiber composites.

### Longitudinal Tensile Strength

We now consider the strength characteristics of continuous and aligned fiber-reinforced composites that are loaded in the longitudinal direction. Under these circumstances, strength is normally taken as the maximum stress on the stress-strain curve, Figure 16.9b; often this point corresponds to fiber fracture, and marks the onset of composite failure. Table 16.1 lists typical longitudinal

**Table 16.1** Typical Longitudinal and Transverse Tensile Strengths for Three Unidirectional Fiber-Reinforced Composites. The Fiber Content for Each Is Approximately 50 Vol%

<i>Material</i>	<i>Longitudinal Tensile Strength (MPa)</i>	<i>Transverse Tensile Strength (MPa)</i>
Glass–polyester	700	20
Carbon (high modulus)–epoxy	1000	35
Kevlar–epoxy	1200	20

**Source:** D. Hull and T. W. Clyne, *An Introduction to Composite Materials*, 2nd edition, Cambridge University Press, New York, 1996, p. 179.

tensile strength values for three common fibrous composites. Failure of this type of composite material is a relatively complex process, and several different failure modes are possible. The mode that operates for a specific composite will depend on fiber and matrix properties, and the nature and strength of the fiber–matrix interfacial bond.

If we assume that  $\epsilon_f^* < \epsilon_m^*$  (Figure 16.9a), which is the usual case, then fibers will fail before the matrix. Once the fibers have fractured, most of the load that was borne by the fibers is now transferred to the matrix. This being the case, it is possible to adapt the expression for the stress on this type of composite, Equation 16.7, into the following expression for the longitudinal strength of the composite,  $\sigma_{cl}^*$ :

$$\sigma_{cl}^* = \sigma_m'(1 - V_f) + \sigma_f^* V_f \quad (16.17)$$

Here  $\sigma_m'$  is the stress in the matrix at fiber failure (as illustrated in Figure 16.9a) and, as previously,  $\sigma_f^*$  is the fiber tensile strength.

### Transverse Tensile Strength

The strengths of continuous and unidirectional fibrous composites are highly anisotropic, and such composites are normally designed to be loaded along the high-strength, longitudinal direction. However, during in-service applications transverse tensile loads may also be present. Under these circumstances, premature failure may result inasmuch as transverse strength is usually extremely low—it sometimes lies below the tensile strength of the matrix. Thus, the reinforcing effect of the fibers is negative. Typical transverse tensile strengths for three unidirectional composites are contained in Table 16.1.

Whereas longitudinal strength is dominated by fiber strength, a variety of factors will have a significant influence on the transverse strength; these factors include properties of both the fiber and matrix, the fiber–matrix bond strength, and the presence of voids. Measures that have been employed to improve the transverse strength of these composites usually involve modifying properties of the matrix.

For a continuous and aligned fiber-reinforced composite, longitudinal strength in tension



### Concept Check 16.2

The following table lists four hypothetical aligned fiber-reinforced composites (labeled A through D), along with their characteristics. On the basis of these data, rank the four composites from highest to lowest strength in the longitudinal direction, and then justify your ranking.

Composite	Fiber Type	Vol. Fraction Fibers	Fiber Strength (MPa)	Ave. Fiber Length (mm)	Critical Length (mm)
A	glass	0.20	$3.5 \times 10^3$	8	0.70
B	glass	0.35	$3.5 \times 10^3$	12	0.75
C	carbon	0.40	$5.5 \times 10^3$	8	0.40
D	carbon	0.30	$5.5 \times 10^3$	8	0.50

[The answer may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site).]

### Discontinuous and Aligned-Fiber Composites

Even though reinforcement efficiency is lower for discontinuous than for continuous fibers, discontinuous and aligned-fiber composites (Figure 16.8b) are becoming increasingly more important in the commercial market. Chopped-glass fibers are used most extensively; however, carbon and aramid discontinuous fibers are also employed. These short-fiber composites can be produced having moduli of elasticity and tensile strengths that approach 90% and 50%, respectively, of their continuous-fiber counterparts.

For a discontinuous and aligned-fiber composite having a uniform distribution of fibers and in which  $l > l_c$ , the longitudinal strength ( $\sigma_{cd}^*$ ) is given by the relationship

$$\sigma_{cd}^* = \sigma_f^* V_f \left( 1 - \frac{l_c}{2l} \right) + \sigma_m' (1 - V_f) \quad (16.18)$$

where  $\sigma_f^*$  and  $\sigma_m'$  represent, respectively, the fracture strength of the fiber and the stress in the matrix when the composite fails (Figure 16.9a).

If the fiber length is less than critical ( $l < l_c$ ), then the longitudinal strength ( $\sigma_{cd}^*$ ) is given by

$$\sigma_{cd}^* = \frac{l\tau_c}{d} V_f + \sigma_m' (1 - V_f) \quad (16.19)$$

where  $d$  is the fiber diameter and  $\tau_c$  is the smaller of either the fiber-matrix bond strength or the matrix shear yield strength.

### Discontinuous and Randomly Oriented-Fiber Composites

Normally, when the fiber orientation is random, short and discontinuous fibers are used; reinforcement of this type is schematically demonstrated in Figure 16.8c. Under these circumstances, a “rule-of-mixtures” expression for the elastic modulus similar to Equation 16.10a may be used, as follows:

$$E_{cd} = KE_f V_f + E_m V_m \quad (16.20)$$

For a discontinuous ( $l > l_c$ ) and aligned fiber-reinforced composite, longitudinal strength in tension

For a discontinuous ( $l < l_c$ ) and aligned fiber-reinforced composite, longitudinal strength in tension

For a discontinuous and randomly oriented fiber-reinforced composite, modulus of elasticity

**Table 16.2** Properties of Unreinforced and Reinforced Polycarbonates with Randomly Oriented Glass Fibers

Property	Unreinforced	Fiber Reinforcement (vol%)		
		20	30	40
Specific gravity	1.19–1.22	1.35	1.43	1.52
Tensile strength [MPa (ksi)]	59–62 (8.5–9.0)	110 (16)	131 (19)	159 (23)
Modulus of elasticity [GPa ( $10^6$ psi)]	2.24–2.345 (0.325–0.340)	5.93 (0.86)	8.62 (1.25)	11.6 (1.68)
Elongation (%)	90–115	4–6	3–5	3–5
Impact strength, notched Izod (lb <sub>f</sub> /in.)	12–16	2.0	2.0	2.5

**Source:** Adapted from Materials Engineering's *Materials Selector*, copyright © Penton/IPC.

In this expression,  $K$  is a fiber efficiency parameter that depends on  $V_f$  and the  $E_f/E_m$  ratio. Of course, its magnitude will be less than unity, usually in the range 0.1 to 0.6. Thus, for random fiber reinforcement (as with oriented), the modulus increases in some proportion of the volume fraction of fiber. Table 16.2, which gives some of the mechanical properties of unreinforced and reinforced polycarbonates for discontinuous and randomly oriented glass fibers, provides an idea of the magnitude of the reinforcement that is possible.

By way of summary, then, aligned fibrous composites are inherently anisotropic in that the maximum strength and reinforcement are achieved along the alignment (longitudinal) direction. In the transverse direction, fiber reinforcement is virtually nonexistent: fracture usually occurs at relatively low tensile stresses. For other stress orientations, composite strength lies between these extremes. The efficiency of fiber reinforcement for several situations is presented in Table 16.3; this efficiency is taken to be unity for an oriented-fiber composite in the alignment direction, and zero perpendicular to it.

When multidirectional stresses are imposed within a single plane, aligned layers that are fastened together one on top of another at different orientations are frequently used. These are termed *laminar composites*, which are discussed in Section 16.14.

**Table 16.3** Reinforcement Efficiency of Fiber-Reinforced Composites for Several Fiber Orientations and at Various Directions of Stress Application

Fiber Orientation	Stress Direction	Reinforcement Efficiency
All fibers parallel	Parallel to fibers	1
	Perpendicular to fibers	0
Fibers randomly and uniformly distributed within a specific plane	Any direction in the plane of the fibers	$\frac{3}{8}$
Fibers randomly and uniformly distributed within three dimensions in space	Any direction	$\frac{1}{5}$

**Source:** H. Krenchel, *Fibre Reinforcement*, Copenhagen: Akademisk Forlag, 1964 [33].

Applications involving totally multidirectional applied stresses normally use discontinuous fibers, which are randomly oriented in the matrix material. Table 16.3 shows that the reinforcement efficiency is only one-fifth that of an aligned composite in the longitudinal direction; however, the mechanical characteristics are isotropic.

Consideration of orientation and fiber length for a particular composite will depend on the level and nature of the applied stress as well as fabrication cost. Production rates for short-fiber composites (both aligned and randomly oriented) are rapid, and intricate shapes can be formed that are not possible with continuous fiber reinforcement. Furthermore, fabrication costs are considerably lower than for continuous and aligned; fabrication techniques applied to short-fiber composite materials include compression, injection, and extrusion molding, which are described for unreinforced polymers in Section 15.22.



### Concept Check 16.3

Cite one desirable characteristic and one less-desirable characteristic for each of (1) discontinuous-oriented and (2) discontinuous-randomly oriented fiber-reinforced composites.

[The answer may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site)]

## 16.6 THE FIBER PHASE

An important characteristic of most materials, especially brittle ones, is that a small-diameter fiber is much stronger than the bulk material. As discussed in Section 12.8, the probability of the presence of a critical surface flaw that can lead to fracture diminishes with decreasing specimen volume, and this feature is used to advantage in the fiber-reinforced composites. Also, the materials used for reinforcing fibers have high tensile strengths.

### whisker

On the basis of diameter and character, fibers are grouped into three different classifications: *whiskers*, *fibers*, and *wires*. **Whiskers** are very thin single crystals that have extremely large length-to-diameter ratios. As a consequence of their small size, they have a high degree of crystalline perfection and are virtually flaw-free, which accounts for their exceptionally high strengths; they are among the strongest known materials. In spite of these high strengths, whiskers are not used extensively as a reinforcement medium because they are extremely expensive. Moreover, it is difficult and often impractical to incorporate whiskers into a matrix. Whisker materials include graphite, silicon carbide, silicon nitride, and aluminum oxide; some mechanical characteristics of these materials are given in Table 16.4.

### fiber

Materials that are classified as **fibers** are either polycrystalline or amorphous and have small diameters; fibrous materials are generally either polymers or ceramics (e.g., the polymer aramids, glass, carbon, boron, aluminum oxide, and silicon carbide). Table 16.4 also presents some data on a few materials that are used in fiber form.

Fine wires have relatively large diameters; typical materials include steel, molybdenum, and tungsten. Wires are used as a radial steel reinforcement in automobile tires, in filament-wound rocket casings, and in wire-wound high-pressure hoses.