İki Boyutlu Projektif Dönüşüm

$$X = \frac{a_{11}x + a_{12}y + a_{13}}{a_{31}x + a_{32}y + 1} \qquad\qquad i = 1, 2, 3, 4$$

$$Y = \frac{a_{21}x + a_{22}y + a_{23}}{a_{31}x + a_{32}y + 1}$$

$$a_{11}x_i + a_{12}y_i + a_{13} + \qquad\qquad - a_{31}x_i X_i - a_{32}y_i X_i = X_i$$

$$a_{21}x_i + a_{22}y_i + a_{23} - a_{31}x_i Y_i - a_{32}y_i Y_i = Y_i$$

Koşul denklemleri, a_{11}, a_{12},.....a_{23} katsayıları 8 tanedir ve bu nedenle 4 ortak noktanın (X,Y) ve (x,y) koordinatları yardımıyla çözülür.

Eğer ortak nokta sayısı 4'den fazla ise bilinmeyenler dengelemeli olarak iteratif çözümle bulunur. İlk olarak,

$$1\text{-} \quad \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2- Düzeltme denklemleri,

$$i = 1, 2, ...n$$

$$N_i = \frac{1}{a_{31}x_i + a_{32}y_i + 1}$$

$$V_{xi} = \frac{x_i}{N_i} da_{11} + \frac{y_i}{N_i} da_{12} + \frac{1}{N_i} da_{13} - \frac{x_i X_i}{N_i} da_{31} - \frac{y_i X_i}{N_i} da_{32} - \frac{l_{xi}}{N_i}$$

$$V_{yi} = \frac{x_i}{N_i} da_{21} + \frac{y_i}{N_i} da_{22} + \frac{1}{N_i} da_{23} - \frac{x_i Y_i}{N_i} da_{31} - \frac{y_i Y_i}{N_i} da_{32} - \frac{l_{yi}}{N_i}$$

$$-l_{xi} = a_{11}x_i + a_{12}y_i + a_{13} - a_{31}x_i X_i - a_{32}y_i X_i - X_i$$

$$-l_{yi} = a_{21}x_i + a_{22}y_i + a_{23} - a_{31}x_i Y_i - a_{32}y_i Y_i - Y_i$$

3- Normal denklemlerin kurulması ve çözümü

$$V = AX - L$$

$$X = (A^T PA)^{-1} A^T PL$$

$$V = \begin{bmatrix} V_{x1} \\ V_{y1} \\ \cdot \\ \cdot \\ V_{xn} \\ V_{yn} \end{bmatrix} \quad , \quad L = \begin{bmatrix} L_{x1} \\ L_{y1} \\ \cdot \\ \cdot \\ L_{xn} \\ L_{yn} \end{bmatrix} \quad , \quad X = \begin{bmatrix} da_{11} \\ da_{12} \\ da_{13} \\ da_{21} \\ da_{22} \\ da_{23} \\ da_{31} \\ da_{32} \end{bmatrix}$$

$$
A = \begin{bmatrix}
\dfrac{x_1}{N_1} & \dfrac{y_1}{N_1} & \dfrac{1}{N_1} & 0 & 0 & 0 & -\dfrac{x_1 X_1}{N_1} & -\dfrac{y_1 X_1}{N_1} \\[2mm]
0 & 0 & 0 & \dfrac{x_1}{N_1} & \dfrac{y_1}{N_1} & \dfrac{1}{N_1} & -\dfrac{x_1 Y_1}{N_1} & -\dfrac{y_1 Y_1}{N_1} \\[2mm]
. & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . \\
\dfrac{x_n}{N_n} & \dfrac{y_n}{N_n} & \dfrac{1}{N_n} & 0 & 0 & 0 & -\dfrac{x_n X_n}{N_n} & -\dfrac{y_n X_n}{N_n} \\[2mm]
0 & 0 & 0 & \dfrac{x_n}{N_n} & \dfrac{y_n}{N_n} & \dfrac{1}{N_n} & -\dfrac{x_n Y_n}{N_n} & -\dfrac{y_n Y_n}{N_n}
\end{bmatrix}
$$

4- İterasyonla çözüm,

$$a_{11}^{k+1} = a_{11}^{k} + da_{11} \quad , \quad a_{12}^{k+1} = a_{12}^{k} + da_{12} \quad , \quad a_{13}^{k+1} = a_{13}^{k} + da_{13}$$

$$a_{21}^{k+1} = a_{21}^{k} + da_{21} \quad , \quad a_{22}^{k+1} = a_{22}^{k} + da_{22} \quad , \quad a_{23}^{k+1} = a_{23}^{k} + da_{23}$$

$$a_{31}^{k+1} = a_{31}^{k} + da_{31} \quad , \quad a_{32}^{k+1} = a_{32}^{k} + da_{32}$$

5- Bilinmeyenler değişmeyene kadar 1-4 işlem adımları tekrarlanır.