



T.C.
ONDOKUZ MAYIS ÜNİVERSİTESİ

VEZİRKÖPRÜ MESLEK YÜKSEKOKULU

ÇOCUK BAKIMI VE GENÇLİK HİZMETLERİ
BÖLÜMÜ/ ÇOCUK GELİŞİMİ PROGRAMI

TEMEL MATEMATİK

ÜSLÜ SAYILAR

10. Hafta

özenilen üniversite

Üslü Sayılar:

$$a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ tane } a \text{ nin çarpımı}}$$

a: taban
n: üs

$$* 3^3 = 3 \cdot 3 \cdot 3 = 27$$

$$* 5^2 = 5 \cdot 5 = 25$$

not: $a + a + a + \dots + a = na$
n tane a'nın toplamı

* Parantez ve İşaretler:

* $(+)^n$: pozitif sayıların tüm kuvvetleri pozitifdir.

* $(-)^{2n}$: parantezin içi negatif üssü çift ise sayı dışarı pozitif çıkar.

* $(-)^{2n+1}$: Parantez içi negatif, üssü tek ise sayı dışarı negatif çıkar.

$$* (5)^3 = 5^3$$

$$* (-5)^3 = -5^3 \text{ (parantez yokken üs, işarete etki edemez)}$$

$$* (-5)^4 = 5^4$$

not: Parantezin içindeki üs, işarete etki etmez.

$$\begin{aligned} (-3)^2 &\neq (-3^2) \\ \downarrow &\quad \downarrow \\ (-3) \cdot (-3) &= 9 \quad -3 \cdot 3 = -9 \end{aligned}$$

$$* a^0 = 1 \quad a \neq 0, \quad 0^0 = \text{tanımsız}$$

$$3^0 = 1 \quad 1^0 = 1 \quad (-1461)^0 = 1$$

$$1791^0 = 1 \quad e^0 = 1 \quad \left(-\frac{2}{13}\right)^0 = 1$$

not: 0, parantezli ifadenin üssü değilse işarete etki etmez.

$$* a \cdot x^n + b \cdot x^n - c \cdot x^n = (a+b-c) \cdot x^n$$

$$* 8^5 + 3 \cdot 8^5 - 4 \cdot 8^5 = (1+3-4) \cdot 8^5 = 0 \cdot 8^5 = 0$$

$$* 4 \cdot 17^3 + 2 \cdot 17^3 = (4+2) \cdot 17^3 = 6 \cdot 17^3$$

$$* a^{-n} = \left(\frac{1}{a}\right)^n, \quad \left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^n$$

$$* 2^{-1} = \left(\frac{1}{2}\right)^1$$

$$* \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$$

$$* 3^{-3} = \left(\frac{1}{3}\right)^3$$

$$* \left(-\frac{2}{3}\right)^{-3} = -\left(\frac{3}{2}\right)^3$$

$$* -3^{-3} = -\left(\frac{1}{3}\right)^3$$

$$* \left(-\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$$

$$a^x \cdot a^y = a^{x+y} \rightarrow 2^3 \cdot 2^5 = 2^8$$

$$\frac{a^x}{a^y} = a^{x-y} \rightarrow \frac{2^3}{2^5} = 2^{3-5} = 2^{-2} = \left(\frac{1}{2}\right)^2$$

$$(a^b)^c = a^{b \cdot c} \rightarrow (-2^3)^2 = (2)^6 = 64$$

$$a^x \cdot b^x = (a \cdot b)^x \rightarrow 12^x = (2 \cdot 2 \cdot 3)^x = 2^x \cdot 2^x \cdot 3^x$$

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x \rightarrow \frac{6^x}{3^x} = \left(\frac{6}{3}\right)^x = 2^x$$

$$a^{\frac{x}{n}} = b^{\frac{y}{n}} \rightarrow 5^1 = 2^x \Rightarrow 5^{\frac{1}{x}} = 2^{\frac{x}{x}} = 2$$

$$\left. \begin{aligned} a^x &= b^y \\ a^z &= b^t \end{aligned} \right\} \frac{x}{z} = \frac{y}{t} \rightarrow \left. \begin{aligned} 2^x &= 3^4 \\ 2^9 &= 3^x \end{aligned} \right\} \frac{x}{9} = \frac{4}{x} \quad \begin{aligned} x^2 &= 36 \\ x &= \pm 6 \end{aligned}$$

2001-ÖSS: $\left[(-\frac{1}{2})^{-3}\right]^2 = ?$

Çözüm: $(\frac{1}{2})^{-6} = (2)^6 = \boxed{64}$

1989-ÖSS: $\left[(-\frac{1}{2})^{-1}\right]^3 = ?$

Çözüm: $(-\frac{1}{2})^{-3} = (-2)^3 = \boxed{-2^3}$

1983-ÖYS: $(\frac{1}{2})^2 : (-\frac{1}{2})^6 = ?$

Çözüm: $\frac{1}{4} : \frac{1}{64} = \frac{1}{4} \cdot \frac{64}{1} = 16 = \boxed{2^4}$

1993-ÖYS: $\frac{3^2 + (-2)^3}{(-1)^4 + 2^2} = ?$

Çözüm: $\frac{9-8}{1+4} = \boxed{\frac{1}{5}}$

1986-ÖSS: $\frac{(-\frac{1}{2})^3 \cdot (-2^4)}{(-2)^2} = ?$

Çözüm: $\frac{-\frac{1}{8} \cdot -16}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

1997-ÖSS: $(2^{-1} + 2^0)^{-2} \cdot 3^2 = ?$

Çözüm: $(\frac{1}{2} + 1)^{-2} \cdot 9 = (\frac{3}{2})^{-2} \cdot 9 = (\frac{2}{3})^2 \cdot 9 = \frac{4}{9} \cdot 9 = \boxed{4}$

2002-ÖSS: $\left(\frac{(\frac{1}{2})^{-1} : (\frac{1}{2})^2}{(\frac{1}{2})^3}\right)^{\frac{1}{2}} = ?$

Çözüm: $(2 : \frac{1}{4} : \frac{1}{8})^{\frac{1}{2}} = (2 \cdot \frac{4}{1} \cdot \frac{8}{1})^{\frac{1}{2}} = 64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} = \boxed{8}$

2004-ÖSS: $(-\frac{1}{8})^{-\frac{2}{3}} = ?$

Çözüm: $(-\frac{8}{1})^{\frac{2}{3}} = [(-2)^3]^{\frac{2}{3}} = (-2)^2 = \boxed{4}$

2011-YGS: $\frac{4^{\frac{1}{2}} + (-8)^{\frac{1}{3}} - 1}{2^{-1}} = ?$

Çözüm: $\frac{(2^2)^{\frac{1}{2}} + [(-2)^3]^{\frac{1}{3}} - 1}{\frac{1}{2}} = \frac{2 + -2 - 1}{\frac{1}{2}} = \frac{-1}{\frac{1}{2}} = \boxed{-2}$

2012-YGS: $\frac{6^{-2} - 4 \cdot 6^{-3}}{3^{-2} - 2 \cdot 3^{-3}} = ?$

Çözüm: $\frac{\frac{1}{36(6)} - \frac{4 \cdot 1}{216}}{\frac{1}{9} - 2 \cdot \frac{1}{27}} = \frac{\frac{6-4}{216}}{\frac{3-2}{27}} = \frac{2}{27} \cdot \frac{27}{1} = \boxed{\frac{1}{4}}$

2005-ÖSS: $\frac{2^{12} + 2^{13}}{2^{14} - 2^{15}} = ?$

Çözüm: $\frac{2^{12}(1+2^1)}{2^{12}(2^2-2^3)} = \frac{3}{4-8} = \boxed{-\frac{3}{4}}$

1993-ÖSS: $\frac{2^{93} - 2^{92}}{2^{94}} = ?$

Çözüm: $\frac{2^{92}(2^1 - 1)}{2^{92} \cdot 2^2} = \boxed{\frac{1}{4}}$

2010-YGS: $15^{13} + 6 \cdot 15^{13} + 8 \cdot 15^{13} = ?$

Çözüm: $(1+6+8) \cdot 15^{13} = 15 \cdot 15^{13} = \boxed{15^{14}}$

1998-ÖYS: $\frac{14^a + 14^a}{7^a + 7^a + 7^a + 7^a} = 32$ ise $a = ?$

Çözüm: $\frac{2 \cdot 14^a}{2 \cdot 7^a} = \frac{2^a \cdot 7^a}{2 \cdot 7^a} = \frac{2^a}{2} = 2^{a-1}$

$2^{a-1} = 32$ ise $a-1=5$, $\boxed{a=6}$

1990-ÖSS: $3(a^2)^3 - 2(a^3)^2 - a^5 = ?$

Çözüm: $3a^6 - 2a^6 - a^5 = a^6 - a^5$

1987-ÖSS: a pozitif bir sayı ise aşağıdakilerden hangisi negatiftir?
A) a^{-2} B) a^{-1} C) $-(-a)^3$ D) $(-a)^2$ E) $-a^{-3}$

Çözüm: a=1 olsun.

$$a^{-2} = 1^{-2} = 1$$

$$a^{-1} = 1^{-1} = 1$$

$$-(-a)^3 = -(-1)^3 = -(-1) = 1$$

$$(-a)^2 = (-1)^2 = 1$$

$$-a^{-3} = -1^{-3} = -1$$

1985-ÖSS: $(-a)^7 \cdot (-a^4) \cdot (-a)^{-2} = ?$

Çözüm: $-a^7 \cdot -a^4 \cdot a^{-2} = +a^{7+4-2} = a^9$

2010-YGS: $(16)^{3n} = 8^5$ ise n=?

Çözüm: $(2^4)^{3n} = (2^3)^5$, $2^{12n} = 2^{15}$

$$12n = 15, \quad n = \frac{15}{12} = \frac{5}{4}, \quad \boxed{n = \frac{5}{4}}$$

2003-ÖSS: $4 - 4^x + 3^x \cdot 4^{x+1} = \frac{48}{12^{1-x}}$, x=?

Çözüm: $4 - 4^x + 3^x \cdot 4^x \cdot 4 = \frac{48^4}{12 \cdot 12^{-x}}$

$$4 - 4^x + 4 \cdot 12^x = 4 \cdot 12^x, \quad 4 = 4^x, \quad \boxed{x=1}$$

1999-ÖSS: $3 \cdot 2^{x+2} + 4 \cdot 2^x = 8$ ise x=?

Çözüm: $3 \cdot 2^x \cdot 2^2 + 4 \cdot 2^x = 8$

$$12 \cdot 2^x + 4 \cdot 2^x = 8, \quad 16 \cdot 2^x = 8$$

$$2^x = \frac{8}{16} = \frac{1}{2} = 2^{-1}$$

$$2^x = 2^{-1}, \quad \boxed{x=-1}$$

1997-ÖSS: $\frac{2^{n+1} + 3^n}{2 \cdot 3^{n-2}} + \frac{2^n - 2^{n-1}}{2^{n-2}} = ?$

Çözüm: $\frac{2^n(2+1)}{2^n \cdot 2 \cdot 3^{n-2}} + \frac{2^n(1-2^{-1})}{2^n \cdot 2^{-2}} = \frac{4}{2} + \frac{1 - \frac{1}{2}}{\frac{1}{4}}$
 $= \frac{4}{1} \cdot \frac{9}{2} + \frac{1}{2} \cdot \frac{4}{1} = 18 + 2 = \boxed{20}$

1987-ÖSS: $2^{x+1} + 6(2^x) + 4(2^{x-1}) = 80$, x=?

Çözüm: $2^x \cdot 2 + 6 \cdot 2^x + 4 \cdot 2^x \cdot 2^{-1} = 80$

$$2^x(2+6+4 \cdot \frac{1}{2}) = 80, \quad 2^x \cdot 10 = 80$$

$$2^x = 8 \quad \text{ise} \quad \boxed{x=3}$$

1996-ÖSS: $54 \cdot 3^x + 3^{x+3} - 729 = 0$, x=?

Çözüm: $54 \cdot 3^x + 3^x \cdot 3^3 = 729$

$$3^x(54+27) = 729, \quad 3^x(81) = 729$$

$$3^x = 9, \quad \boxed{x=2}$$

2009-ÖSS: $3^m = 2$ ise $3^{2m+1} = ?$

Çözüm: $3^{2m+1} = 3^m \cdot 3^m \cdot 3^1 = 2 \cdot 2 \cdot 3 = \boxed{12}$

2001-ÖSS: $x > 0$, $a = 2^x$ ise

$$\frac{4^{x+1} - 4}{2^{x+1} - 2} = ?$$

Çözüm: $2^x = a$ ise $2^x \cdot 2^x = 4^x = a^2$

$$\frac{4^x \cdot 4 - 4}{2^x \cdot 2 - 2} = \frac{4a^2 - 4}{2a - 2} = \frac{4(a^2 - 1)}{2(a - 1)} = \frac{2(a-1)(a+1)}{(a-1)} = \boxed{2(a+1)}$$

2001-ÖSS: $3^m = a$, $7^m = b$ ise $(147)^m = ?$

Çözüm: $(147)^m = (7 \cdot 7 \cdot 3)^m = 7^m \cdot 7^m \cdot 3^m = \boxed{a \cdot b^2}$

1996-ÖSS: $2^x = a$, $3^x = b$ ise $72^x = ?$

Çözüm: $72^x = (2 \cdot 2 \cdot 2 \cdot 3 \cdot 3)^x = 2^x \cdot 2^x \cdot 2^x \cdot 3^x \cdot 3^x = \boxed{a^3 \cdot b^2}$

2013-YGS: $\frac{2^{-2}}{\frac{1}{4} + \frac{1}{m^{-1}}} = 13^{-1}$ ise $m = ?$

Çözüm: $\frac{\frac{1}{4}}{\frac{1}{4} + \frac{m}{(4)}} = \frac{1}{13}$, $\frac{\frac{1}{4}}{\frac{1+4m}{4}} = \frac{1}{13}$

$\frac{1}{1+4m} \times \frac{1}{13}$, $13 = 1+4m$, $12=4m$, $m=3$

2013-YGS: $2^x = 6^{x+y-1}$ ise $3^x = ?$

Çözüm: $2^x = 6^x \cdot 6^y \cdot 6^{-1}$
 $2^x = 2^x \cdot 3^x \cdot 6^y \cdot 6^{-1}$
 $\frac{1}{6^y \cdot 6^{-1}} = 3^x$, $3^x = 6^{1-y}$

2012-LYS: $(\sqrt{7}+\sqrt{3})^x = 4$ ise $(\sqrt{7}-\sqrt{3})^x = ?$

Çözüm: $(\sqrt{7}+\sqrt{3})^x = 4$
 $\times (\sqrt{7}-\sqrt{3})^x = A$
 $4^x = 4A$, $A = \frac{4^x}{4}$, $A = 4^{x-1}$

2012-LYS: $\frac{3^x}{2^{2x}} = \frac{1}{5}$ ise $5^{\frac{1}{x}} = ?$

Çözüm: $\frac{3^x}{(2^2)^x} = \frac{1}{5}$, $\frac{3^x}{4^x} = \frac{1}{5}$, $\frac{5}{1} = \frac{4^x}{3^x}$
 $5^1 = \left(\frac{4}{3}\right)^x \rightarrow 5^{\frac{1}{x}} = \left(\frac{4}{3}\right)^{\frac{x}{x}} = \frac{4}{3}$

2012-YGS: $2^x - 2^{-y} (2^{x+y} - 2) = ?$

Çözüm: $2^x - 2^{x+y-y} + 2^{-y+1} = 2^{-y+1}$

2011-YGS: $12^a = 2$, $6^b = 3$ ise $12^{(1-a) \cdot 2b} = ?$

Çözüm: $12^{1-a} = 12 \cdot 12^{-a} = \frac{12}{12^a} = \frac{12}{2} = 6$

$12^{(1-a) \cdot 2b} = 6^{2b} = 6^b \cdot 6^b = 3 \cdot 3 = 9$

2011-YGS: $\frac{x^2-y^2}{4x^2+xy} = \frac{1}{2}$ ise $(x+y)^2 = ?$

Çözüm: $\frac{x^2-y^2}{(2^2)x^2+xy} = \frac{1}{2}$, $\frac{x^2-y^2-2x^2-2xy}{(2^2)x^2+xy} = \frac{-1}{2}$

$\Rightarrow -x^2-2xy-y^2 = -1 \Rightarrow x^2+2xy+y^2 = 1$

$(x+y)^2 = 1$

2009-ÖSS: $3^x = 5^y$, $3^z = 5^t$ ise x, y, z, t arasındaki bağıntı nedir?

Çözüm: $\begin{cases} 3^x = 5^y \\ 3^z = 5^t \end{cases} \Rightarrow \frac{x}{z} = \frac{y}{t}$, $xt = yz$

2007-ÖSS: $\frac{2^x - 2 \cdot 3^{x+y} + 3^{2y}}{3^{2x} - 3^{x+y}} = ?$

Çözüm: $3^x = a$, $3^y = b$ olsun.

$\frac{a^2 - 2ab + b^2}{a^2 - ab} = \frac{(a-b)^2}{a(a-b)} = \frac{a-b}{a} = 1 - \frac{b}{a}$

$\Rightarrow 1 - \frac{3^y}{3^x} = 1 - 3^{y-x}$

2005-ÖSS: $2^a = 3$, $3^b = 4$, $4^c = 8$ ise $a \cdot b \cdot c = ?$

Çözüm:

$\begin{cases} 2^a = 3 \\ 2^2 = 3^b \end{cases} \Rightarrow \frac{a}{2} = \frac{1}{b} \rightarrow a \cdot b = 2$
 $\begin{cases} 4^c = 8 \\ 2^{2c} = 2^3 \end{cases} \Rightarrow 2c = 3, c = \frac{3}{2}$
 $a \cdot b \cdot c = 2 \cdot \frac{3}{2} = 3$

2004-ÖSS: $x \neq 1$, $2^{2x+y} - 2^{x+y+1} - 2^{x+2} = 0$
 x ile y arasındaki bağıntı nedir?

Çözüm: $2^{x+y}(2^x - 2) - (2^x - 2) = 0$
 $(2^x - 2) \cdot (2^{x+y} - 1) = 0$, $2^x = 2^1$, $2^{x+y} = 1$
 $x \neq 1 \Rightarrow x+y=0$

1995-ÖSS: $2^{a-1}=4$ ise $4^{a-1}=?$

Çözüm: $2^a \cdot 2^{-1}=4$, $2^a \cdot \frac{1}{2}=4$, $2^a=8$

$4^{a-1} = \frac{4^a}{4} = \frac{2^a \cdot 2^a}{4} = \frac{8 \cdot 8}{4} = 16$

1993-ÖSS: $5^x=4$ ise $(125)^x + 5^{x+2}=?$

Çözüm:

$(125)^x + 5^{x+2} = (5 \cdot 5 \cdot 5)^x + 5^x \cdot 5^2 = 5^x \cdot 5^x \cdot 5^x + 5^x \cdot 25$
 $= 4 \cdot 4 \cdot 4 + 4 \cdot 25 = 64 + 100 = 164$

1994-ÖSS: $6^{x+1}=3^{x+2}$ ise $2^{x+1}=?$

Çözüm: $2^{x+1} \cdot 3^{x+1} = 3^{x+1} \cdot 3^1$, $2^{x+1}=3$

1977-ÜSS: $y=2^x$ ise $2^{x+3}=?$

Çözüm: $2^{x+3} = 2^x \cdot 2^3 = y \cdot 8 = 8y$

1982-ÖSS: $\frac{1}{a^{x-y}+1} + \frac{1}{a^{y-x}+1}=?$

Çözüm:

$\frac{1}{\frac{a^x}{a^y}+1} + \frac{1}{\frac{a^y}{a^x}+1} = \frac{1}{\frac{a^x+a^y}{a^y}} + \frac{1}{\frac{a^y+a^x}{a^x}}$
 $= \frac{a^y+a^x}{a^x+a^y} = 1$

1982-ÖSS: $\left(\frac{a^x}{a^y}\right)^{x-y} \cdot \left(\frac{a^y}{a^x}\right)^{x-y}=?$

Çözüm:

$\left[\frac{a^x}{a^y} \cdot \frac{a^y}{a^x}\right]^{x-y} = 1^{x-y} = 1$

1994-ÖSS: $m, n \in \mathbb{Z}$, $\left(\frac{1}{n}\right)^m = 8$ ise

$m+n=?$

Çözüm: $\left(\frac{1}{2}\right)^{-3}$ ise $m+n=2-3=-1$

$\left(\frac{1}{8}\right)^{-1}$ ise $m+n=8-1=7$

* Bu soruda $m+n$ kaç olabilir diye sorulmıyordu...

Gök büyük ve Gök küçük Sayılar

$10^3 = 1000$

$12 \cdot 10^7 = 120000000$

$5 \cdot 10^{-3} = 0,005$

$0,16 \cdot 10^{-3} = 1,6 \cdot 10^{-4} = 16 \cdot 10^{-5}$

2003-ÖSS: $\frac{(0,005 \cdot 10^{35}) + (0,8 \cdot 10^{23})}{10^{32}}=?$

Çözüm: $\frac{5 \cdot 10^{32} + 8 \cdot 10^{32}}{10^{32}} = \frac{13 \cdot 10^{32}}{10^{32}} = 13$

1996-ÖSS: $\left(\frac{0,018}{0,006}\right)^{a+1} = (27)^{1-a}$, $a=?$

Çözüm: $\left(\frac{18 \cdot 10^{-3}}{6 \cdot 10^{-3}}\right)^{a+1} = (3^3)^{1-a}$

$3^{a+1} = 3^{3-3a}$, $a+1=3-3a$, $4a=2$, $a=\frac{1}{2}$

1995-ÖSS: $(0,027)^{\frac{5}{3}} \cdot 10^5=?$

Çözüm: $(27 \cdot 10^{-3})^{\frac{5}{3}} \cdot 10^5 = (3^3)^{\frac{5}{3}} \cdot (10^{-3})^{\frac{5}{3}} \cdot 10^5$
 $= 3^5 \cdot 10^{-5} \cdot 10^5 = 3^5$

1994-ÖSS: $\frac{4 \cdot 10^{-3} + 3 \cdot 10^{-4}}{10^{-4}}=?$

Çözüm: $\frac{40 \cdot 10^{-4} + 3 \cdot 10^{-4}}{10^{-4}} = \frac{43 \cdot 10^{-4}}{10^{-4}} = 43$

1982-ÖSS: $5 \cdot (0,03)^3=?$

Çözüm: $5 \cdot (3 \cdot 10^{-2})^3 = 5 \cdot 3^3 \cdot 10^{-6} = 135 \cdot 10^{-6}$

Basamak Sayısı:

$10^3 = 1+3=4$ basamak

$3 \cdot 10^4 = 1+4=5$ basamak

$124 \cdot 10^7 = 3+7=10$ basamak

Örnek: $3 \cdot 2^7 \cdot 5^6$ kaç basamaklıdır?

Çözüm: $3 \cdot 2 \cdot 2^6 \cdot 5^6 = 6 \cdot 10^6$

$1+6=7$ basamak

Örnek: $3^x = 16$, $8^y = 9$ ise $x \cdot y = ?$

Çözüm: $\left. \begin{array}{l} 3^x = 2^4 \\ 3^2 = 2^{3y} \end{array} \right\} \frac{x}{2} = \frac{4}{3y}, 3xy = 8$
 $\boxed{x \cdot y = \frac{8}{3}}$

Örnek: $\left. \begin{array}{l} 8^a = 27 \\ 81 = 4^b \end{array} \right\}$ ise $\frac{a+b}{b+2a} = ?$

Çözüm: $\left. \begin{array}{l} 2^{3a} = 3^3 \\ 2^b = 3^4 \end{array} \right\} \frac{3a}{2b} = \frac{3}{4}, 2a = b$

$$\frac{a+b}{b+2a} = \frac{a+2a}{2a+2a} = \frac{3a}{4a} = \boxed{\frac{3}{4}}$$

Örnek: $32^a = 9$, $27 = 16^b$ ise $\frac{a}{b} = ?$

Çözüm: $\left. \begin{array}{l} 2^{5a} = 3^2 \\ 2^{4b} = 3^3 \end{array} \right\} \frac{5a}{4b} = \frac{2}{3}, \frac{a}{b} = \frac{8}{15}$

Sıralama

Sıralama yapılırken üsler ya da tabanlar eşitlenir.

1996-ÖYS:

$$x = (2^3)^4, y = 2^{(3^4)}, z = (2^{12})^3 \text{ ise}$$

x, y, z yi sıralayın?

Çözüm:

$$x = 2^{12}, y = 2^{81}, z = 2^{36}, \boxed{y > z > x}$$

Örnek: $\left. \begin{array}{l} a = 8^5 \\ b = 16^3 \\ c = 32^2 \end{array} \right\}$ ise a, b, c yi sıralayın?

Çözüm: $\left. \begin{array}{l} a = (2^3)^5 = 2^{15} \\ b = (2^4)^3 = 2^{12} \\ c = (2^5)^2 = 2^{10} \end{array} \right\} a > b > c$

Örnek: $\left. \begin{array}{l} a = 5^{90} \\ b = 16^{36} \\ c = 9^{54} \end{array} \right\}$ ise a, b, c yi sıralayın?

Çözüm:

$$\begin{array}{ccc|c} 90 & 144 & 108 & 9 \\ 10 & 16 & 12 & 2 \\ 5 & 8 & 6 & \end{array}$$

$a = 5^{90}$
 $b = (2^4)^{36} = 2^{144}$
 $c = (3^2)^{54} = 3^{108}$
 OBE (90, 144, 108) = 18

$$\left. \begin{array}{l} a = (5^5)^{18} = 3125^{18} \\ b = (2^8)^{18} = 256^{18} \\ c = (3^6)^{18} = 729^{18} \end{array} \right\} \boxed{a > c > b}$$

Örnek:

$\left. \begin{array}{l} a = 5^{80} \\ b = 4^{100} \\ c = 9^{60} \end{array} \right\}$ ise a, b, c yi sıralayın?

Çözüm:

$$\begin{array}{ccc|c} 80 & 100 & 120 & 40 \\ 5 & 4 & 9 & \end{array}$$

$a = 5^{80}$
 $b = 2^{200}$
 $c = 3^{120}$
 OBE (80, 200, 120) = 40

$$\left. \begin{array}{l} a = (5^2)^{40} = 25^{40} \\ b = (2^5)^{40} = 32^{40} \\ c = (3^3)^{40} = 27^{40} \end{array} \right\} \boxed{b > c > a}$$

Örnek:

$2^a = 39$
 $3^b = 85$ ise a, b, c yi sıralayın?
 $5^c = 129$

Çözüm:

$$\begin{array}{l} 2^5 < 39 < 2^6 \rightarrow 2^5 < 2^a < 2^6 \rightarrow 5 < a < 6 \\ 3^4 < 85 < 3^5 \rightarrow 3^4 < 3^b < 3^5 \rightarrow 4 < b < 5 \\ 5^3 < 129 < 5^4 \rightarrow 5^3 < 5^c < 5^4 \rightarrow 3 < c < 4 \end{array}$$

$$\boxed{a > b > c}$$

Not: $a^x = b^y$ ise $x=y=0$ olmalı.

Not: $a^x = a^y$ ise $a \neq -1, 0, 1$ için $x=y$ dir.

Not: $a^b = 1$ ise,
 • $b=0$ ve $a \neq 0$
 • $a=1$
 • $a=-1$ ve b çift

Not: $a^x = b^x$ ise, x tek ise $a=b$
 x çift ise $a=\pm b$

Örnek: $4^{x-y+3} = 3^{2x+y-12}$ ise $x, y = ?$

Çözüm:

Dereceler sıfır olmalıdır.

$$\begin{array}{r} x-y = -3 \\ + 2x+y = 12 \\ \hline 3x = 9, x=3, y=6, \boxed{x \cdot y = 18} \end{array}$$

Örnek: $(x-4)^{8-2x} = 1$ ise x kaç olabilir?

Çözüm:

- $8-2x=0 \Rightarrow x=4$ (Tabanı 0 yapıyor)
- $x-4=1 \Rightarrow \boxed{x=5}$
- $x-4=-1 \Rightarrow \boxed{x=3}$ (Dereceyi çift yapar)

$$\text{C.K.} = \{3, 5\}$$

Örnek: $(a+3)^{a^2+3a} = 1$ ise $a, k = ?$

Çözüm:

- $a^2+3a=0$ ise $\boxed{a=0}$
 $a=-3$ (tabanı 0 yapıyor)
- $a+3=1 \Rightarrow \boxed{a=-2}$
- $a+3=-1 \Rightarrow \boxed{a=-4}$ (dereceyi çift yapar)

$$\text{C.K.} = \{-4, -2, 0\}$$

Örnek: $(3x-5)^{2007} = (x+7)^{2007}$ ise x kaçtır?

Çözüm: $3x-5=x+7, 2x=12, \boxed{x=6}$

Örnek: $(3x-5)^{2012} = (x+7)^{2012}$, $C.K. = ?$

Çözüm: $3x-5=x+7 \vee 3x-5=-x-7$
 $2x=12 \quad 4x=-2$
 $\boxed{x=6} \quad \vee \quad \boxed{x=-\frac{1}{2}}$

$$\text{C.K.} = \left\{ -\frac{1}{2}, 6 \right\}$$

Not: Eşitsizliklerde tabanın mutlak değeri daima 1 den büyük yapılırsa üstlerde yön değişimi yapılmaz!..

2008-ÖSS: $3^{4-x} \leq 1 \leq 5^{6-x}$, x tam sayılarının toplamı kaçtır?

Çözüm: $3^{4-x} \leq 1 \quad 1 \leq 5^{6-x}$
 $\downarrow \quad \downarrow$
 $3^{4-x} \leq 3^0 \quad 5^0 \leq 5^{6-x}$
 $4-x \leq 0 \quad 0 \leq 6-x$
 $\boxed{4 \leq x} \quad \boxed{x \leq 6}$
 $4 \leq x \leq 6 \rightarrow 4+5+6 = \boxed{15}$

2006-ÖSS: $4^{2m-1} > \left(\frac{1}{16}\right)^{m+7}$ ise en küçük m tam sayısı kaçtır?

Çözüm: $4^{2m-1} > (4^{-2})^{m+7}$
 $4^{2m-1} > 4^{-2m-14} \rightarrow 2m-1 > -2m-14$
 $4m > -13$
 \downarrow
 $\boxed{m=-3}$

KAYNAKLAR

Matematik Ders Notları - Ayşe Yıldırım & Ceyhan Yavuz
<http://calameo.download/003359034eff296032d3c> (Eriřim 23.08.2018)

http://www.buders.com/DOKUMAN/bukaynak/matematik_calisma_sorulari/denklem_kurma_problemleri_calisma_kagidi_1.pdf (Eriřim 14.04.2017)

Temel Matematik – Basri ÇELİK – Dora Basım Yayın